24th COBEM - 2017

24th ABCM International Congress of Mechanical Engineering
December 3-8, 2017, Curitiba, PR, Brazil

COBEM-2017-1612
INVERTED PENDULUM DAMPER DESIGN PARAMETERS FOR VIBRATION CONTROL IN TALL BUILDINGS

P. L. Bernardes Junior
Universidade de Brasília, Programa de Pós-Graduação em Integridade de Materiais da Engenharia, Brasilia, Brazil
pedro.bernardes.enc@gmail.com

M.V.G. Morais
mumorais@gmail.com

S. M. Avila
avilas@unb.br

Abstract. *The constant progress of constructive techniques and civil construction materials, coupled with technological advancement, especially in the field of computational simulations, has allowed the design of increasingly tall and slender structures. These structures presents high flexibility and therefore are vulnerable to dynamic loads actions, such as earthquakes, winds and even occupation by human users or machines. Due to this flexibility, these structures commonly suffer from excessive vibration, which can cause users discomfort or compromise the building safety. A passive vibration control system can be resumed as one or more devices that absorbs a portion of the energy transmitted by external loads. One of these devices is the tuned mass damper (TMD), which transfers energy between the main structure and an auxiliary mass. One of the most used Tuned Mass Dampers in building structures are the conventional pendulum types, but they have the inconvenience of requiring a lot of internal space inside the building. In this work a ten-storey shear-building model is analyzed by reducing it to a single degree of freedom system in order to present a preliminary set of design parameters for an Inverted Pendulum Tuned Mass Damper (IPTMD), which can be used to protect dynamically loaded structures against undesirable vibration levels.*

Keywords: *structural control, tuned mass damper, structural dynamics, inverted pendulum*

1. INTRODUCTION

The structural control is a technique that promotes a change on the stiffness and damping properties of a structure. It can be classified as passive, active, hybrid or semi-active (Soong & Dargush, 1997). A passive control system can be resumed as the installation of one or more devices at the main structure in order to absorb a portion of the energy transmitted by external dynamic loads (Avila, 2002 and Deraemaeker et al., 2016).

One typical passive control device is the Tuned Mass Damper (TMD), which transfers energy between the main structure and an auxiliary mass. It is possible to design a tuned mass damper (TMD) by the simplified analysis of the structure as a discrete model. The TMD attached to the main structure needs to be tuned to a particular frequency in order to make it vibrate out of phase with the main system, absorbing energy from the structure and minimizing the main system’s vibration. Thus, when optimally tuned to the resonant mode of interest, a TMD commonly is effective in reducing the dynamic response of the structure.

A TMD may be designed with a pendulum geometry. The most used Pendulum TMD’s in building structures are conventional pendulums, but they have the inconvenience of, sometimes, require a lot of internal space in the building. Therefore, the inverted pendulum becomes an adequate alternative (Guimarães, 2016). On the other hand, according to Anh et al. (2007), to study and design an inverted-pendulum system is a big challenge compared to conventional pendulum-type ones, as the first can be stable or unstable while the latter is mostly stable.

In this work, a ten-storey shear-building model is analyzed. It is reduced to a single degree of freedom system, in order to determine a set of design parameters for an Inverted Pendulum Tuned Mass Damper (IPTMD).

2. REDUCED SCALE PHYSICAL MODEL

To simulate the behavior of a tall building, the parameters and characteristics of a reduced scale physical model was used. This model consists of ten SAE 1020 steel modules, with Young's Module equals to 2.05x1011 and specific weight equals to 7870 Kg/m³. The modules are bolted one by one and each module is set by two plates connected by four columns. Plates and columns have both a thickness equals to 6,3 mm. The resulting structure is approximately tall 212 centimeters, with an approximately slenderness ratio of 1:10.

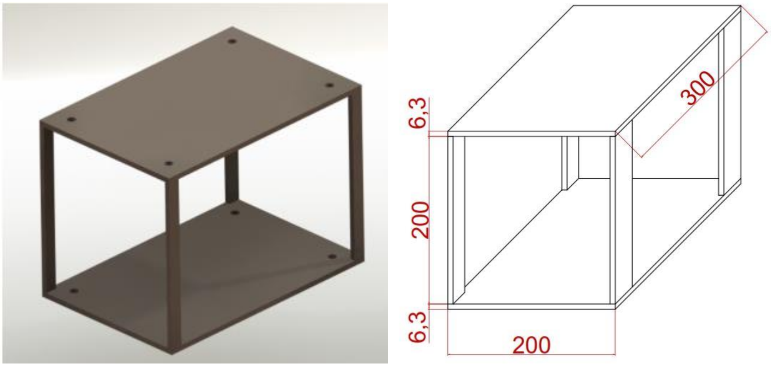

Figure 1. Steel Module used to assemble the physical model (a) and module dimensions (mm) (b)

Figure 2. Resulting structure assembled

3. SHEAR FRAME MODEL

In order to perform the necessary analysis, the physical model was analyzed as a ten-storey shear frame, shown in Figure 3. The shear frame model is a simplification that assumes that all of the building mass is lumped at the floor levels, that the floor beams have infinite stiffness and not considering columns axial effect. Together, these assumptions allow to assume that the displacements at each floor may be described by one single translational degree of freedom. Thus, only ten degrees of freedom are needed to describe the displacements of the structure.

The mass of each floor was considered as the mass of the plates on that level, plus the mass of bolts and nuts, subtracting the mass relative to the holes. The stiffness of each floor was assumed to be the sum of all columns stiffness in the considered direction. The stiffness of each column, assumed to be fixed at both joints, is

$$K = \frac{12 \cdot E \cdot I}{L^3} \tag{1}$$

where E is the Young's Modulus of material and I is the cross section's Moment of Inertia at the considered direction.

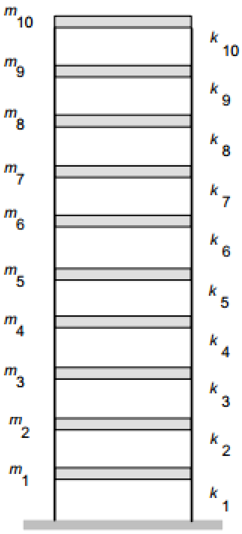

Figure 3. Ten degrees of freedom shear frame

The properties of mass and stiffness of the structure and the resulting natural frequencies are shown in Tables 1 and 2, respectively. The first five modal shapes are illustrated in Figure 4.

Table 1. Stiffness and Mass Properties of the structure.

n	Stiffness k_n (N/m)	Mass m_n (Kg)
1	500901.8720	5.9827
2	592060.4851	6.0086
3	513323.2916	6.0455
4	510294.3134	6.0225
5	502349.2988	6.0224
6	477039.3438	5.9779
7	507729.2016	5.9794
8	481335.9660	6.0016
9	540480.7883	5.9679
10	510765.8977	2.9235

Table 2. Natural Frequencies.

Mode	Natural Frequency (Hz)
1	7.3488
2	21.6324
3	35.4883
4	48.4065
5	60.6354
6	70.4973
7	79.7474
8	86.3231
9	91.2525
10	92.7778

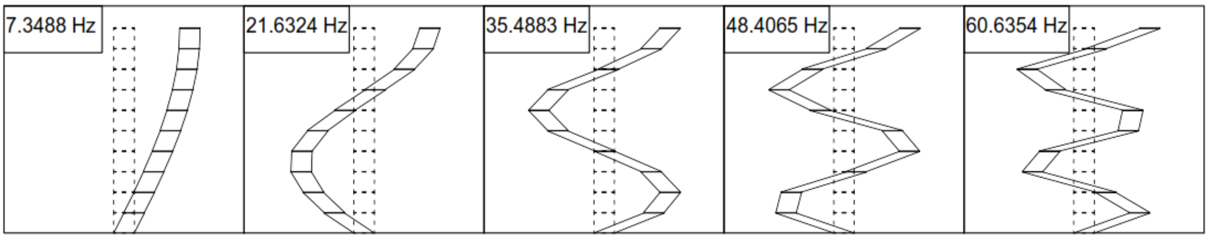

Figure 4. First five modal shapes of the considered shear frame

4. REDUCTION TO A SINGLE DEGREE OF FREEDOM SYSTEM

The equations of motion of the system is:

$$\mathrm{M} \cdot \ddot{y}(t) + \mathrm{C} \cdot \dot{y}(t) + \mathrm{K} \cdot y(t) = \mathrm{F}(t) \tag{2}$$

Where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{F}(t)$ is the dynamic load vector and $\boldsymbol{y}(t)$ is the displacement vector of the structure.

The structural response for systems with many degrees of freedom such as tall buildings, can be obtained through a reduced model using modal analysis (Soong and Dargush, 1997).

Then, we can approximate the dynamic response vector $\boldsymbol{y}(t)$, representing it by a single coordinate $\boldsymbol{y}_N$ and a single mode $\boldsymbol{\phi}_1$

$$y = \phi_1 \cdot y_N(t) \tag{3}$$

Substituting equation (3) into equation (2) and pre-multiplying (2) by Φ_1^T the equation of movement (2) becomes:

$$M_1 \cdot \ddot{y}_N(t) + C_1 \cdot \dot{y}_N(t) + K_1 \cdot y_N(t) = F(t) \tag{4}$$

where $M_1 = \boldsymbol{\Phi}_1^T \mathbf{M} \boldsymbol{\Phi}_1$ is the modal mass, $C_1 = M_1 2\xi_1\omega_1^2$, $K_1 = M_1\omega_1^2$.; ξ_1 and ω_1 are the damping ratio and the natural frequency of the mode. It can be seen that the system with many degrees of freedom is reduced to an equivalent single degree of freedom system, except for the fact that mass, stiffness and modal damping are used instead of the real physical parameters.

With this modal reduction technique and considering ξ_1=0.02, the equivalent single degree of freedom model properties are given: M_1 = 29.5044 Kg; K_1 = 6.2905x10^4 N/m and C_1 = 54.4936 Ns/m.

5. IPTMD DESIGN

The reduced single degree of freedom with an IPTMD attached is shown in Fig.5. To design IPTMD parameters initially, the damping of the main system is set to zero to use the classical optimal expressions for the system parameters presented by Den Hartog (1956), considering the structure as undamped and subjected to an harmonic excitation:

$$\alpha_{opt} = \frac{1}{1+\mu} \tag{5}$$

$$\xi_{d\,opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{6}$$

$$M_d = \mu \cdot M_1 \tag{7}$$

Then the parameters can be calculated by

$$\omega_d = \alpha \cdot \omega_1 \tag{8}$$

$$K_d = \omega_d^{\ 2} \cdot M_d \tag{9}$$

$$C_d = 2M_d \cdot \xi_d \cdot \omega_d \tag{10}$$

where μ is the mass ratio, α is the optimal frequency ratio and ξ_d is the damping ratio of the damper.

The mass ratio μ is commonly assumed between 0.05 and 0.1. According to Oliveira (2012), increasing the mass ratio of the optimized damper, the peak response value tends to be reduced to an asymptotic value. Adopting μ = 0.1 it is possible to have a good design parameter for different damping ratios of the structure. The following values were found: M_d = 2.9504 Kg; ω_d = 41.9765 rad/s (6.6808 Hz); ξ_d = 0.1846; K_d = 5.1988x103 N/m and C_d = 45.7343 Ns/m.

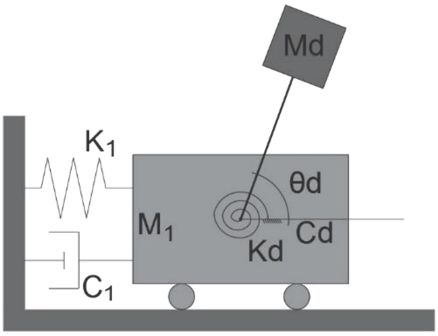

Figure 5. Two degree of freedom main system with IPTMD

Through the equation (11), that describes the natural frequency of the inverted pendulum damper, it is possible to find the optimum length of the rigid bar, setting a value for linear density.

$$\omega_d = \sqrt{\left[\frac{6K_d - gL(6M_d + 3\rho L)}{2L^2(3M_d + \rho L)}\right]} \qquad \text{(Anh et al, 2007) (11)}$$

Considering ρ=0.5423, which is the linear density value of an aluminum bar with a diameter of 16 millimeters, the optimum length obtained is L=0.97 m.

6. RESPONSE OF CONTROLLED SYSTEM

The equations of motion of the damped two degree of freedom of Fig. 5 considering small displacements in the pendulum, when M1 is subjected to an harmonic force F(t), are described in the matrix form in (12).

$$\begin{bmatrix} \frac{\rho L^3}{3} + M_d L^2 & M_d L + \frac{\rho L^2}{2} \\ M_d L + \frac{\rho L^2}{2} & M_1 + M_d + \rho L \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} C_d & 0 \\ 0 & C_1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} K_d - M_d gL - \frac{\rho g L^2}{2} & 0 \\ 0 & K_1 \end{bmatrix} \cdot \begin{bmatrix} \theta \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \end{bmatrix} \tag{12}$$

Where M1, K1 e C1 are the modal mass, stiffness and damping coefficient. Md, Kd e Cd are the IPTMD's mass, stiffness and damping coefficient. ρ and L are the pendulum bar's linear density and length. By making $F(t)=e^{i\omega t}$, $u(t)=H_u(\omega)e^{i\omega t}$ and $\theta(t)= H_\theta(\omega)e^{i\omega t}$ and replacing in equation (12), it can be obtained:

$$\begin{bmatrix} -\left(\frac{\rho L^3}{3} + M_d L^2\right)\omega^2 + C_d i\omega + \left(K_d - M_d gL - \frac{\rho g L^2}{2}\right) & -\left(M_d L + \frac{\rho L^2}{2}\right)\omega^2 \\ -\left(M_d L + \frac{\rho L^2}{2}\right)\omega^2 & -(M_1 + M_d + \rho L)\omega^2 + C_1 i\omega + K_1 \end{bmatrix} \cdot \begin{bmatrix} H_\theta(\omega) \\ H_u(\omega) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{13}$$

Solving the linear system, it is possible to obtain the response functions in the frequency domain $H_u(\omega)$ and $H_\theta(\omega)$:

$$H_u(\omega) = \frac{\omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 + i\omega^3 A_3 + \omega^2 A_2 + i\omega A_1 + A_0} \tag{14}$$

where

$$B_0 = 6(\rho g L^2 + 2M_d gL - 2K_d);$$

$$B_1 = -12C_d;$$

$$B_2 = 4\left(\rho L^3 + 3M_d L^2\right);$$

$$A_0 = 6(-2K_1K_d + K_1 g\rho L^2 + 2K_1 M_d gL);$$

$$A_1 = 6(-2C_1K_d - 2C_dK_1 + 2C_1M_d gL + C_1\rho L^2 g);$$

$$A_2 = 12M_1K_d + 12M_dK_d + 12\rho LK_d + 12M_dK_1L^2 + 4\rho L^3K_1 - 12C_1C_d i^2 - 12M_d{}^2 gL - 6\rho^2L^3g - 12M_1M_d gL - 6M_1\rho L^2 g - 18M_d\rho L^2 g);$$

$$A_3 = 4(3C_dM_1 + 3C_dM_d + 3C_d\rho L + 3C_1M_dL^2 + C_1\rho L^3);$$

$$A_4 = -\rho^2L^4 - 12M_1M_dL^2 - 4M_1\rho L^3 - 4M_d\rho L^3;$$

and

$$H_\theta(\omega) = \frac{\omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 + i\omega^3 A_3 + \omega^2 A_2 + i\omega A_1 + A_0} \tag{15}$$

where

$$B_0 = 0;$$

$$B_1 = 0;$$

$$B_2 = -6\left(\rho L^2 - 2M_d L\right);$$

$$A_0 = 6(-2K_1K_d + K_1 g\rho\rho^2 + 2K_1M_d gL);$$

$$A_1 = 6(-2C_1K_d - 2C_dK_1 + 2C_1M_d gL + C_1\rho L^2 g);$$

$$A_2 = 12M_1K_d + 12M_dK_d + 12\rho LK_d + 12M_dK_1L^2 + 4\rho L^3K_1 - 12C_1C_d i^2 - 12M_d{}^2 gL - 6\rho^2L^3g - 12M_1M_d gL - 6M_1\rho L^2 g - 18M_d\rho L^2 g);$$

$$A_3 = 4(3C_dM_1 + 3C_dM_d + 3C_d\rho L + 3C_1M_dL^2 + C_1\rho L^3);$$

$$A_4 = -\rho^2L^4 - 12M_1M_dL^2 - 4M_1\rho L^3 - 4M_d\rho L^3;$$

Finally, the frequency response function of the structure, with the IPTMD attached is illustrated in Figure 6, where the comparison between the structure with and without control is presented. It can be observed that there is a great reduction on the main system response with the attachment of the IPTMD. The set of parameters defined from Den Hartog's optimal equations was responsible for the reduction in response by more than 85%, although these equations were defined for a mass-spring system, which has a behavior significantly different from a pendulum damper.

It is known that the response of the system is substantially dependent on the length of the pendulum. From the optimal equations and the values found for M_d, K_d, μ and ω_d, as well as the value set for ρ, the required pendulum length was found L=97 cm and the presented response is for this value. However, a pendulum of this length may be difficult to construct, taking into account the overall height of the structure and the possible buckling and flexing effects of the bar.

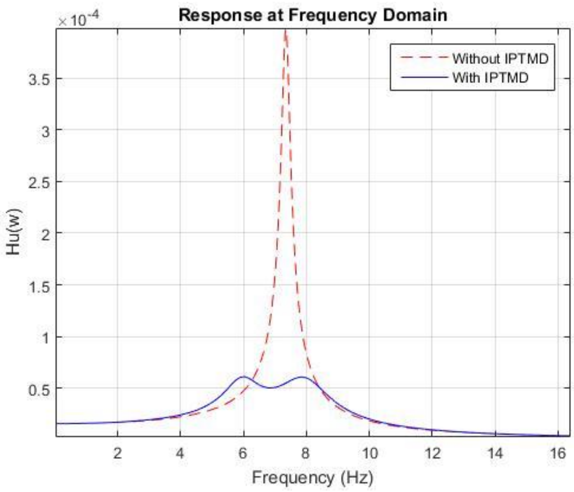

Figure 6. Frequency response of the main system with and without control

7. CONCLUSIONS

This work presents a preliminary design study to set parameters values for an IPTMD. A small scale physical model was considered and it was simplified to a ten storey shear frame. Then the system was reduced to single degree of freedom. Considering the damping of the structure equals to zero, it was possible to use Den Hartog's expressions deduced to a traditional tuned mass damper. Since the damping level in building structures is generally low it showed to be a good preliminary approximation leading to a very good response reduction. This study will be the basis to the design of an IPTMD for the small scale physical model.

8. ACKNOWLEDGEMENTS

This work was supported by the Coordination for the Improvement of Higher Education Personnel (CAPES) and the Federal District Foundation for Research Support (FAPDF).

9. REFERENCES

Anh, N. D., Matsuhisa, H., Vieta, L. D., Yasuda, M., 2007. Vibration Control of an Inverted Pendulum Type Structure by Passive Mass-Spring-Pendulum Dynamic Vibration Absorber. *Journal of Sound and Vibration*, vol. 307, pp. 187-201

Avila, S. M., 2002. *Controle Híbrido para atenuação de vibrações em edifícios*, Tese de doutorado, Pontíficia Universidade Católica do Rio de Janeiro.

Colherinhas, G. B., Morais, M. V. G., Shzu, M. A. M., & Ávila, S. M., 2015. Genetic Optimization of Tower Vibrations with Pendulum TMD. *XXXVI Ibero-Latin American Congress on Computational Methods in Engineering.*

Den Hartog, J. P., 1956. *Mechanical Vibrations*. McGraw-Hill.

Deraemaeker, A., Soltani, P., 2016. A short note on equal peak design for the pendulum tuned mass dampers. *J Multi-body Dynamics*

Guimarães, P. V. B., 2016. Controle Semiativo de Modelo de Pêndulo Invertido para Aerogeradores Off-Shore. Dissertação de Mestrado, Universidade de Brasília.

Oliveira, F. S., 2012. Critérios de Projeto para Amortecedor Tipo Pêndulo para Controle de Vibrações em Edifícios Altos. Dissertação de Mestrado, Universidade de Brasília.

Sherif, A., al-Handasah, D., 2010. Structural Design of Reinforced Concrete Tall Buildings. *Council on Tall Buildings and Urban Habitat Journal*, pp. 34-41.

Soong, T. T., Dargush, G. F., 1997. Passive Energy Dissipation Systems in Structural Engineering. John Wiley & Sons.

Villaverde, R., Koyama, L. A., 1993. Damped Resonant Appendages to Increase Inherent Damping in Buildings. *Earthquake Engineering and Structural Dynamics*, Vol. 22, pp. 491-507.

Zuluaga, A., L., 2007. *Controle de Vibrações em Edifícios Submetidos à Ação de Cargas Dinâmicas Utilizando Amortecedor de Massa Sintonizado na Forma de Pêndulo*. Dissertação de Mestrado, Universidade de Brasília.

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.