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LINEAR STABILITY ANALYSIS OF THE DOUBLE TSUJI BURNER

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Abstract. *Diffusion flames are sensitive to the flow field and, conversely, the flow field is influenced by heat sources – one of the main causes of hydrodynamic instability. In practice, hydrodynamic instability can be either detrimental or beneficial: in an industrial context, for example, hydrodynamic instabilities can lead to harmful conditions for equipment if not correctly predicted and controlled. In contrast, hydrodynamic instabilities generally enhance properties that depend on the contact area or local gradients, such as mixing and reaction rates, which ultimately leads to turbulence. Therefore, the analysis of the stability of chemically reacting flows is crucial for the development of combustion systems. In this analysis, an infinitely long cylindrical burner is assumed, which ejects gaseous fuels radially and uniformly into the ambient. The burner is located in the centre of the impinging flows that transport gaseous oxidiser from the ambient atmosphere to the burner. This configuration is called the double Tsuji burner, derived from the classic Tsuji burner. The main feature of the flame established in this setup – i.e., the double Tsuji flame – is the representation of different flame regimes, from the counterflow regime (close to the vertical axis of symmetry) to the coflow regime (close to the horizontal axis of symmetry), with continuous variations between them. The aim of this work is to determine the hydrodynamic stability of the flow generated by the double Tsuji flame using a (local) linear stability analysis, which provides physical insights and also serves as a starting point for more detailed analyses. The study shows that the flow is stable in all cases investigated, which is physically consistent with the conditions considered.*

Keywords: *hydrodynamic stability, linear stability analysis, interaction between radial and impinging flows, double Tsuji burner, double Tsuji flame*

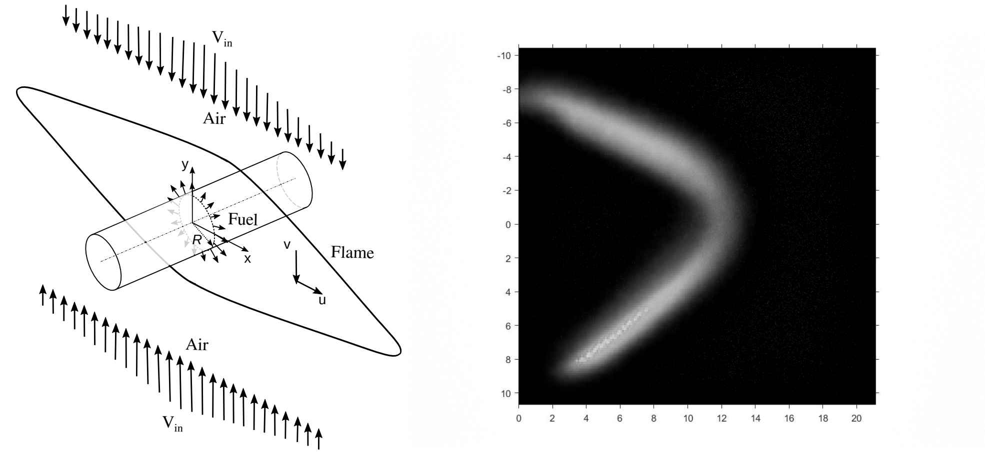
1. INTRODUCTION

The double Tsuji burner is a new configuration for combustion systems (Severino, 2020; Severino *et al.*, 2021, 2022; Li *et al.*, 2022). A distinguishing feature is the provision of flames that vary continuously between counterflow – in the vertical axis of symmetry – and coflow – from the tangency point to the flame tip (Severino *et al.*, 2022), as shown in Fig. 1a.

Under the assumptions adopted in the initial studies, the problem could be seen as the burning of an electrical wire insulator under microgravity conditions such as those found on the International Space Station. (Severino *et al.*, 2021, 2022). However, the proposed configuration also has potential for industrial application as it presents low soot emission. In fact, preliminary experimental results indicate low soot emission, as can be seen in Fig. 1b. Nevertheless, this hypothesis should be investigated further.

In an industrial context, combustion should take place in a specific, ideally small area, as size and material are limited by physical and financial factors. Therefore, an increase in the mixing rate between fuel and oxidiser is necessary, as the molecular mixing process is the main limitation of diffusional combustion (Zeldovich *et al.*, 1985; Williams, 1965; Matalon, 2009). This mixing is strongly dependent on the contact area and the gradients between the reactants. Therefore, it is possible to improve the mixing rate between fuel and oxidiser by introducing entrainment associated with the presence of vortices that lead to the formation of instabilities. Furthermore, the occurrence of instabilities provides additional heat to the load. It should also be noted that, in industrial applications, diffusion flames are preferable for safety reasons (no risk of explosion due to flashbacks) and because of the uniformity of the thermal flux (Baukal, 2000, 2003).

The aim of this work is to determine the stability conditions for the double Tsuji burner in a simplified analysis scope.



(a) (b)

Figure 1: The double Tsuji flame: (a) schematic representation; (b) experimental result.

2. METHODS

In this first study of hydrodynamic stability of the double Tsuji burner, a simplified local linear stability analysis is performed using normal modes (Mendonça, 2000; Yaglom, 2012). The influence of the flame is implicitly introduced by the basic state velocity. Basic state profiles are extracted from previous simulations (Severino *et al.*, 2022) of the double Tsuji flame under incompressible Navier–Stokes flow, mixture fraction and excess enthalpy description (Liñán, 1991, 2001; Fachini, 2007).

2.1 Mathematical model

An incompressible flow is mathematically represented by a (dimensionless) PDE system consisting of an equation for the mass balance and an equation for the momentum balance (Von Mises and Friedrichs, 1971), respectively:

$$\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 \tag{1}$$

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\, \boldsymbol{v} = -\boldsymbol{\nabla} p + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{v} \tag{2}$$

in which, the symbol ∂_t denotes partial time derivatives.

Note that dimensional or non-normalised and dimensionless or normalised quantities are referred to as hat (e.g., $\hat{m}$) and plain (e.g., m) letters respectively. Also, bold (e.g., $\boldsymbol{e}$) and non-bold (e.g., e) symbols are reserved for vectors and scalars in that order.

The dimensionless independent variables are time (t), and the first (x) and second (y) spatial coordinates:

$$t := \frac{\hat{t}}{(1/\hat{a})}, \quad x := \frac{\hat{x}}{\hat{R}}, \quad y := \frac{\hat{y}}{\hat{R}}$$

in which, $\hat{a}$ is the counterflow strain rate and $\hat{R}$ is the burner radius.

The dimensionless dependent variables are the velocity ($\boldsymbol{v}$) and pressure (p):

$$\boldsymbol{v} := \frac{\hat{\boldsymbol{v}}}{\hat{a}\hat{R}}, \quad p := \frac{\hat{p}}{\hat{\rho}_\infty (\hat{a}\hat{R})^2}$$

in which, $\hat{\boldsymbol{v}} := (\hat{u}, \hat{v})$ and $\hat{\rho}$ is the density (the subscript "∞" stands for ambient atmosphere conditions). The Reynolds number (Re) is defined as $\mathrm{Re} := (\hat{a}\hat{R}^2)/(\hat{\nu})$, in which $\hat{\nu}$ is the kinematic viscosity.

2.1.1 Linear stability analysis

By introducing first-order disturbances into the system, the dependent variables can be expressed by

$$f(t,x,y) = f_0(x,y) + \varepsilon f_1(t,x,y) + \mathcal{O}(\varepsilon^2) \tag{3}$$

in which $\varepsilon \ll 1$, and the sub-indices "0" and "1" refer, respectively, to the leading- and first-order terms of each expansion (f is a general representation of u, v and p), which are considered sufficiently smooth.

The evolution of the disturbances can then be determined by comparing the basic (f_0) and disturbed ($f_0 + \varepsilon f_1$) states, both of which satisfy the balance equations. Thus, introducing Eq. (3) into Eqs. (1) and (2):

$$\boldsymbol{\nabla} \cdot \boldsymbol{v}_1 = 0 \tag{4}$$

$$\partial_t \boldsymbol{v}_1 + (\boldsymbol{v}_0 \cdot \boldsymbol{\nabla})\, \boldsymbol{v}_1 + (\boldsymbol{v}_1 \cdot \boldsymbol{\nabla})\, \boldsymbol{v}_0 = -\boldsymbol{\nabla} p_1 + \frac{1}{\mathrm{Re}} \nabla^2 \boldsymbol{v}_1 \tag{5}$$

It is important to note that the parallel flow hypothesis cannot be admitted due to the nature of the system. Therefore, additional terms appear in Eq. (5), compared to the classical analyses (Mendonça, 2000).

The configuration under investigation indeed requires a global analysis (Gennaro and Rodríguez, 2016). In the present work, however, an analysis that is as simplified as possible (local linear analysis) is carried out in order to gain a physical understanding as a first step.

When performing such a modal analysis, the disturbances are described by

$$f_1(t,x,y) = F_1(y) \exp\left(i\alpha x - i\omega t\right) \tag{6}$$

and, substituting Eq. (6) into Eqs. (4) and (5), it results in

Mass balance equation

$$i\alpha U_1 + \frac{\partial V_1}{\partial y} = 0 \tag{7}$$

Momentum balance ($\boldsymbol{x}$-direction)

$$-(i\omega)U_1 + u_0(i\alpha)U_1 + v_0\frac{\partial U_1}{\partial y} + U_1\frac{\partial u_0}{\partial x} + V_1\frac{\partial u_0}{\partial y} = -(i\alpha)P_1 + \frac{1}{\mathrm{Re}}\left(-\alpha^2 U_1 + \frac{\partial^2 U_1}{\partial y^2}\right) \tag{8}$$

Momentum balance ($\boldsymbol{y}$-direction)

$$-(i\omega)V_1 + u_0(i\alpha)V_1 + v_0\frac{\partial V_1}{\partial y} + U_1\frac{\partial v_0}{\partial x} + V_1\frac{\partial v_0}{\partial y} = -\frac{\partial P_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(-\alpha^2 V_1 + \frac{\partial^2 V_1}{\partial y^2}\right) \tag{9}$$

2.1.2 Boundary conditions

Derived from Severino *et al.* (2022), the proposed system for disturbances behaviour is represented by the following boundary conditions:

on the horizontal symmetry axis ($y = 0$),

$$U_1 = \frac{\partial V_1}{\partial y} = 0; \tag{10}$$

and, at the inlet ($y = y_{BC}$),

$$U_1 = V_1 = 0 \tag{11}$$

in which y_{BC} is the domain size in the y-direction (Severino *et al.*, 2022).

Therefore, the hydrodynamic stability problem was converted into a generalised eigenvalue problem.

2.1.3 Spatial and temporal formulations

The terms α and ω are complex values. By making their real and imaginary parts explicit, one has

$$f_1(t,x,y) = F_1(y)\exp\left(-\alpha_i x + \omega_i t\right)\exp\left(i\alpha_r x - i\omega_r t\right) \tag{12}$$

in which, the indices i and r indicate the imaginary and real parts, respectively.

For *spatial formulation*, one takes $\omega_i = 0$ ($\omega = \omega_r$), i.e., ω is real. Thus the disturbance amplitude is varied in the direction of the basic flow, with spatial amplification rate α_i. The frequency is given by ω_r, and the wave number, by α_r.

For *time formulation*, one takes $\alpha_i = 0$ ($\alpha = \alpha_r$), i.e., α is real. Thus, the disturbance changes its amplitude with time, with temporal amplification rate ω_i. In short (Mendonça, 2000):

Spatial analysis formulation

$$\begin{array}{lcccc} \alpha_i < 0 & \rightarrow & \text{amplitude increment} & \rightarrow & \text{instability} \\ \alpha_i = 0 & \rightarrow & \text{amplitude unchanged} & \rightarrow & \text{neutrality} \\ \alpha_i > 0 & \rightarrow & \text{amplitude decrement} & \rightarrow & \text{stability} \end{array} \tag{13}$$

Temporal analysis formulation

$$\begin{array}{lcccc} \omega_i < 0 & \rightarrow & \text{amplitude decrement} & \rightarrow & \text{stability} \\ \omega_i = 0 & \rightarrow & \text{amplitude unchanged} & \rightarrow & \text{neutrality} \\ \omega_i > 0 & \rightarrow & \text{amplitude increment} & \rightarrow & \text{instability} \end{array} \tag{14}$$

2.2 Solution method

Spatial and temporal analyses are performed for the four main cases considered in Severino *et al.* (2022). Local analysis covers the two different regimes: counterflow ($0 \leq x < x_T$) and coflow ($x_T \leq x \leq x_{f,0}$), in which x_T is the first coordinate of the tangency point between the flame and streamlines, (x_T, y_T); and $x_{f,0}$ is the first coordinate of $(x_{f,0}, 0)$, i.e., the point where the flame crosses the x-axis (flame length).

2.2.1 Numerical implementation

The components of the basic flow velocity and their derivatives (sixth-order accuracy) are extracted from previous simulation data (Severino *et al.*, 2022) by cubic spline interpolation, i.e., a piecewise polynomial function is obtained for these variables. The *hydrostab* (Ye *et al.*, 2016b) is used to solve the linearised system via GNU Octave (Eaton *et al.*, 2022). It is a universal solver for hydrodynamic stability problems developed by Han-Yu Ye, Li-jun Yang and Qing-fei Fu from Beihang University, Beijing, China. The code supports two-dimensional problems (in Cartesian coordinates) by performing a normal mode analysis of the linearised equations and boundary conditions. Spectral collocation methods are used for spatial discretisation due to their high accuracy. For more details, see Ye *et al.* (2016a).

3. RESULTS

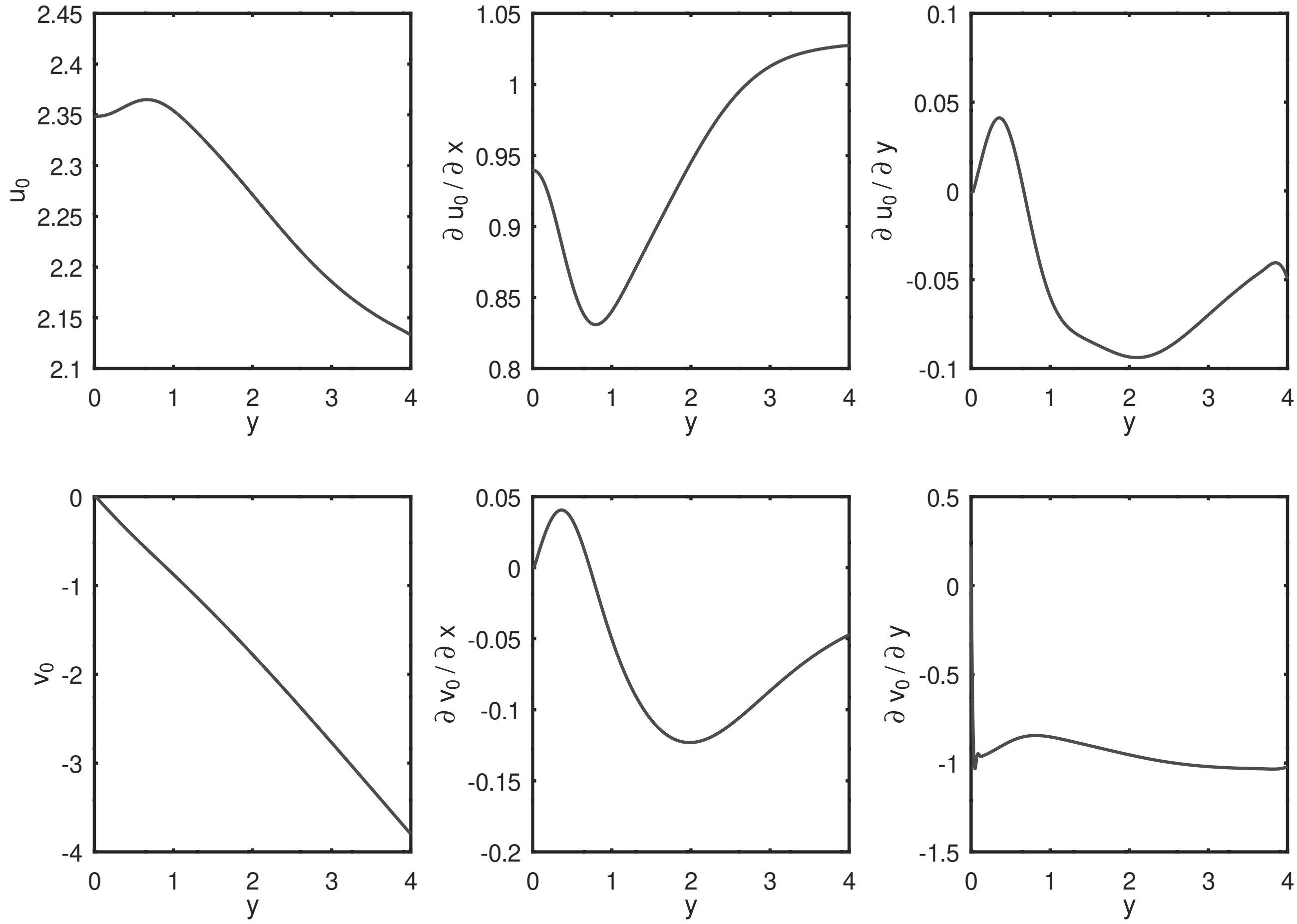
The profiles of u_0, $\partial u_0/\partial x$ and $\partial u_0/\partial y$ (first row), and v_0, $\partial v_0/\partial x$ and $\partial v_0/\partial y$ (second row) for the counterflow (a) and coflow (b) regimes are shown in Figs. 2, 3, 4 and 5. These figures correspond, respectively, to the cases $(S, Pe_b, Pe_c) = (17,1,1)$, $(17,1,17)$, $(1,17,17)$ and $(1,17,1)$, in which S is a chemical parameter, Pe_b is the Péclet number based on burner conditions and Pe_c is the Péclet number based on counterflow conditions. Further details can be found in Severino *et al.* (2021, 2022). For each case, the counterflow regime is considered at $x = 2$, while the coflow regime is considered at $x = x_{f,0}$, in which $x_{f,0} = 36.32, 22.35, 9.31$ and 37.09, respectively.

Linear stability analyses, both spatial and temporal, showed the stability of the double Tsuji burner under the assumptions considered in this work. In addition to the $x = a$ ($a \in [0, x_{f,0}]$) stations mentioned above, some others were considered and also showed decay of the amplitude disturbances and consequent return to the basic state, i.e., stability.

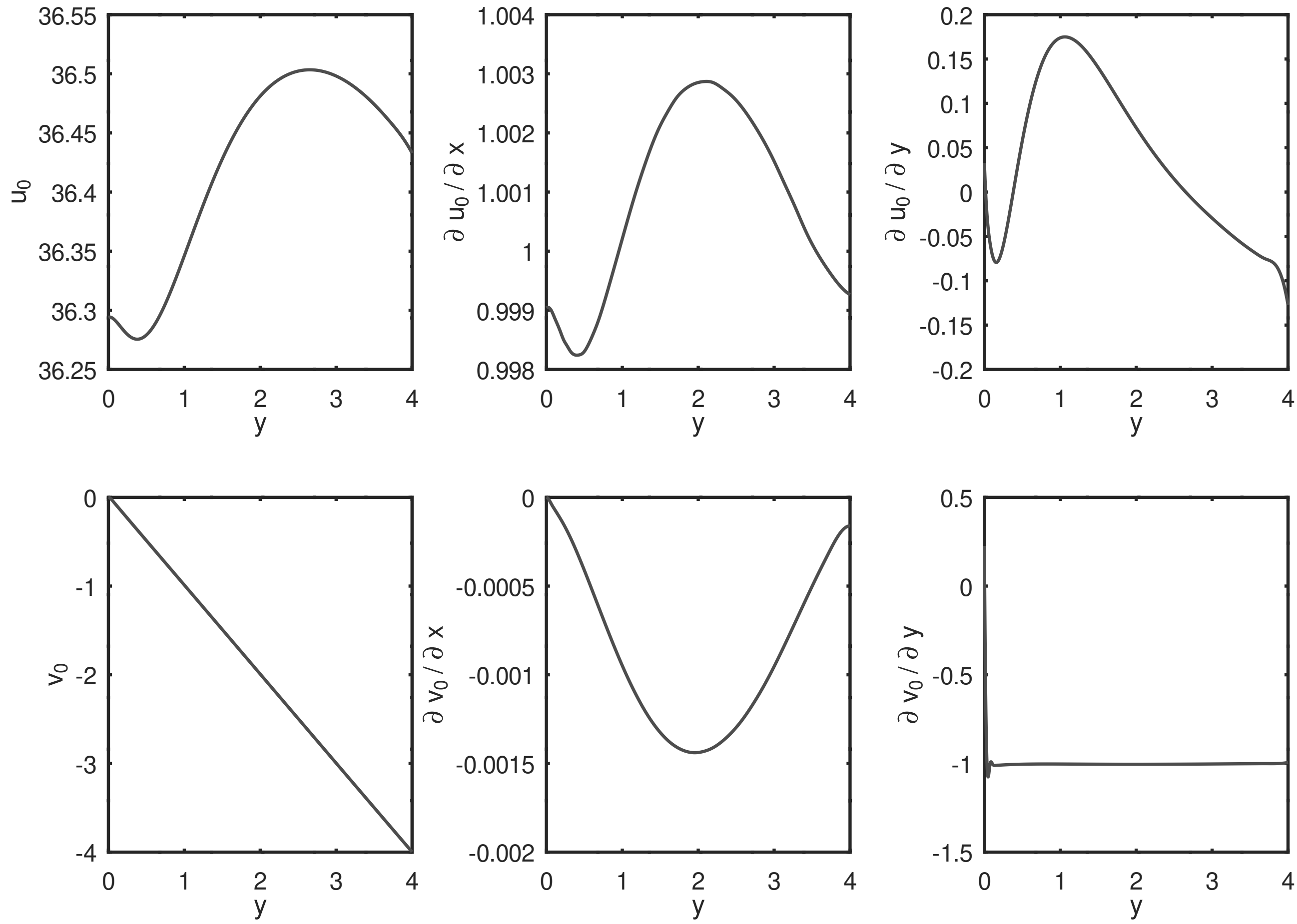
The results found here are physically consistent, since the flow is under the hypothesis of constant density, at low Reynolds number and free of body forces.

4. CONCLUSION

Spatial and temporal linear stability analyses were performed for the double Tsuji burner configuration, considering the four main cases from a previous study. In all cases, the disturbances showed an exponential decay, with the flow returning to the basic state. This makes the double Tsuji burner a stable system under the conditions analysed here. Note that it was expected, given the low Reynolds number, the absence of body forces and the constant density hypothesis. Future analyses will relax these assumptions. In fact, a global analysis will be carried out.

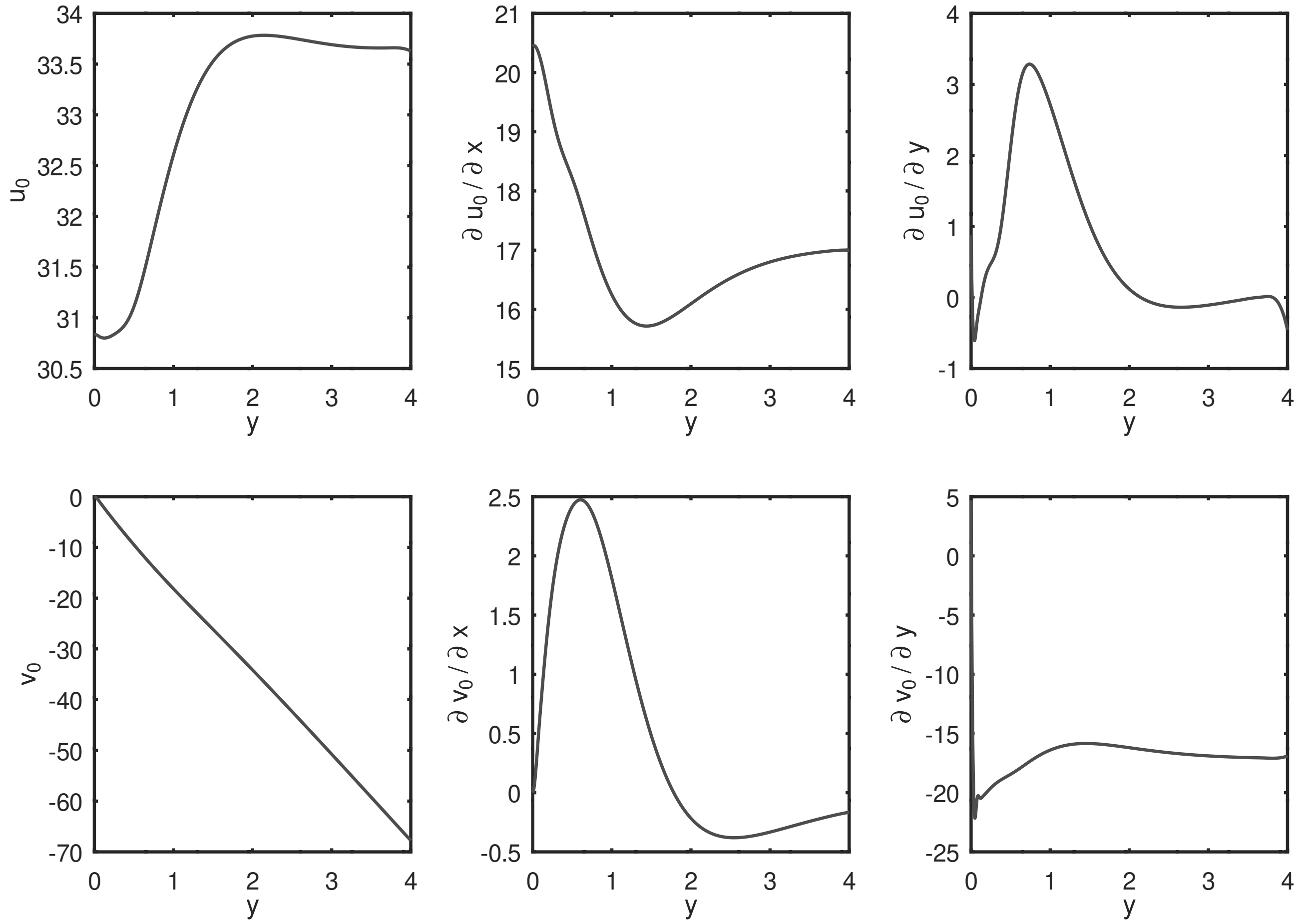


(a)

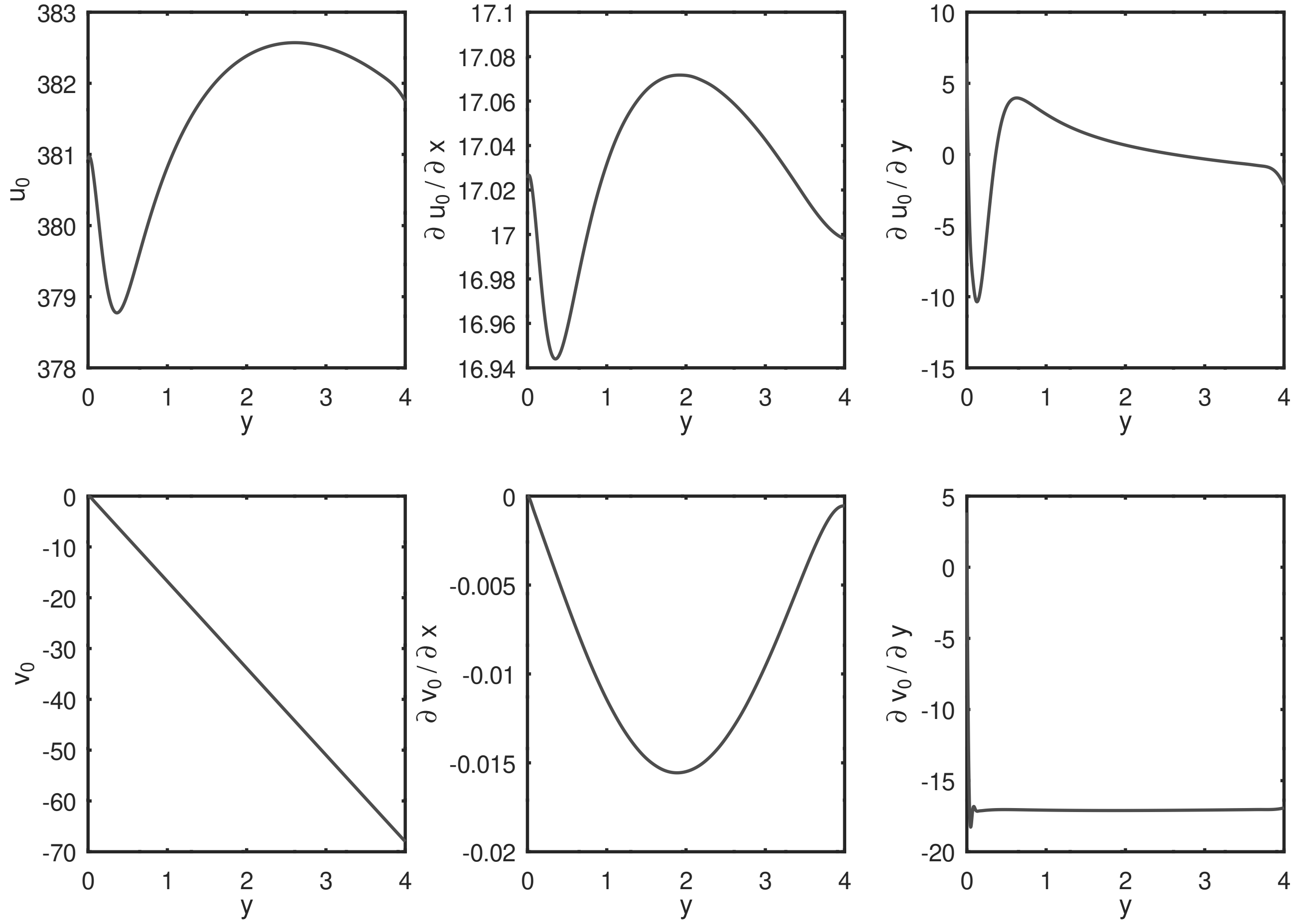


(b)

Figure 2: Counterflow (a) and coflow (b) profiles for the case $(S, Pe_b, Pe_c) = (17, 1, 1)$; (a) $x = 2$ and (b) $x = 36.32$.

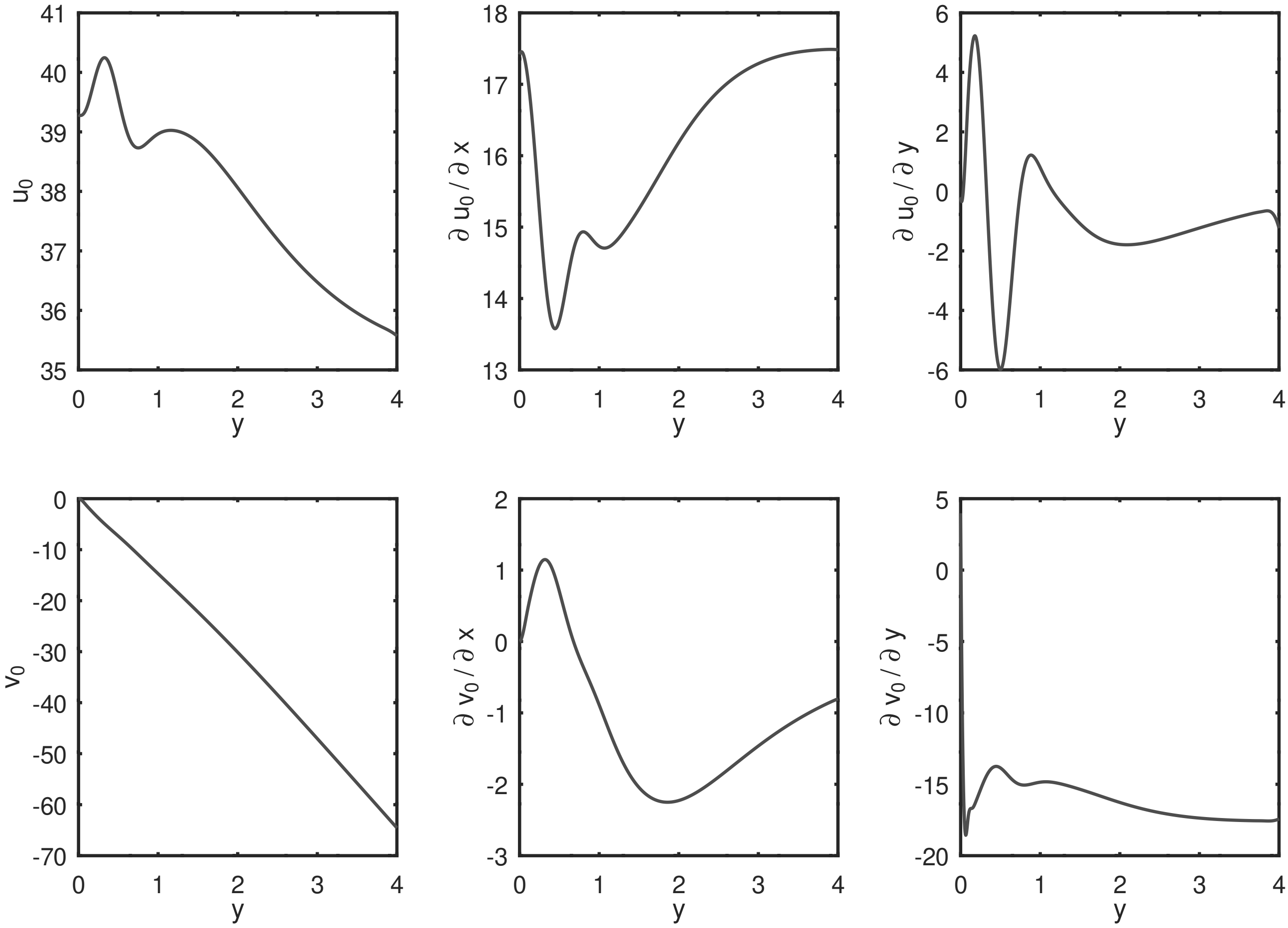


(a)

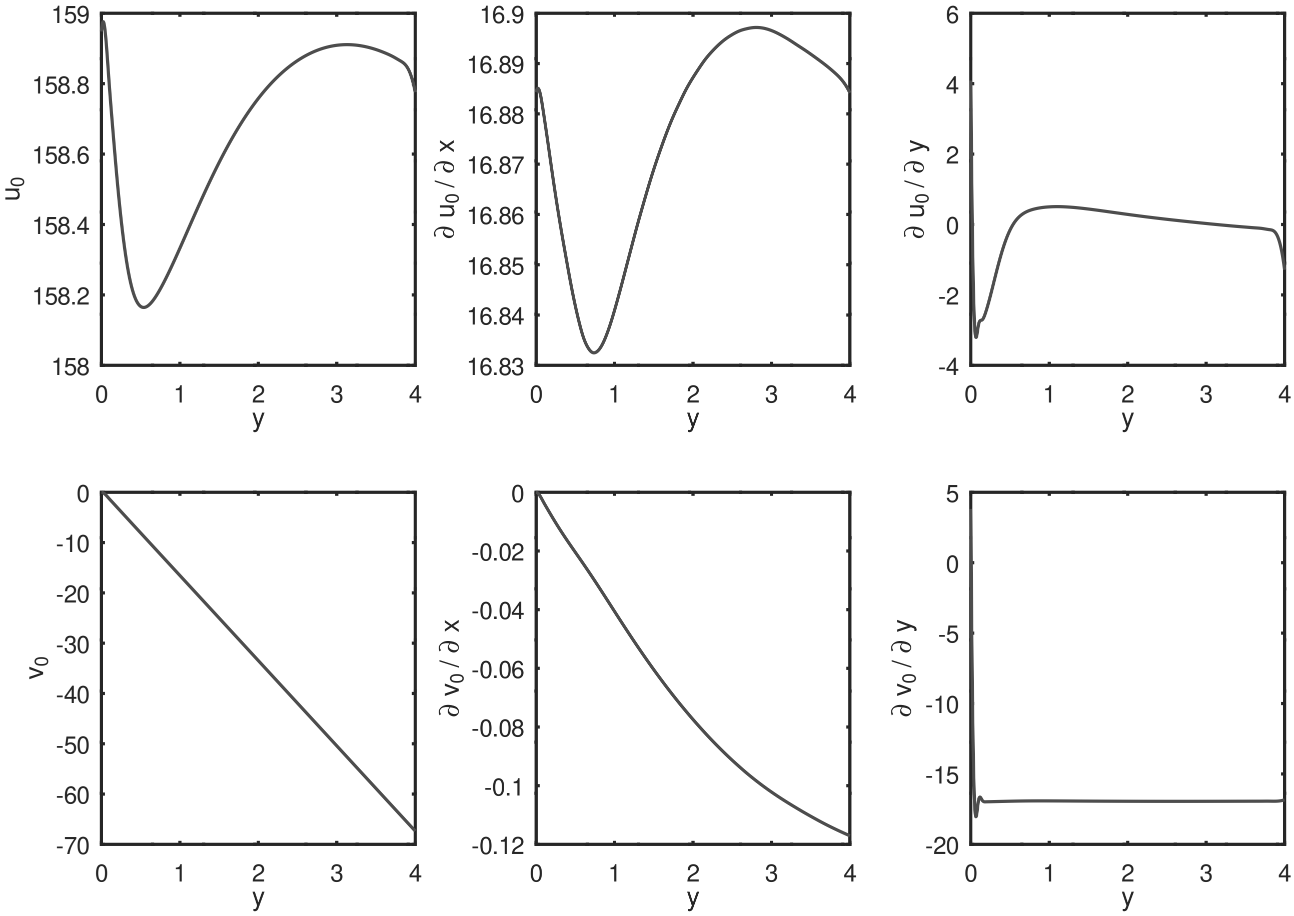


(b)

Figure 3: Counterflow (a) and coflow (b) profiles for the case $(S, Pe_b, Pe_c) = (17, 1, 17)$; (a) $x = 2$ and (b) $x = 22.35$.

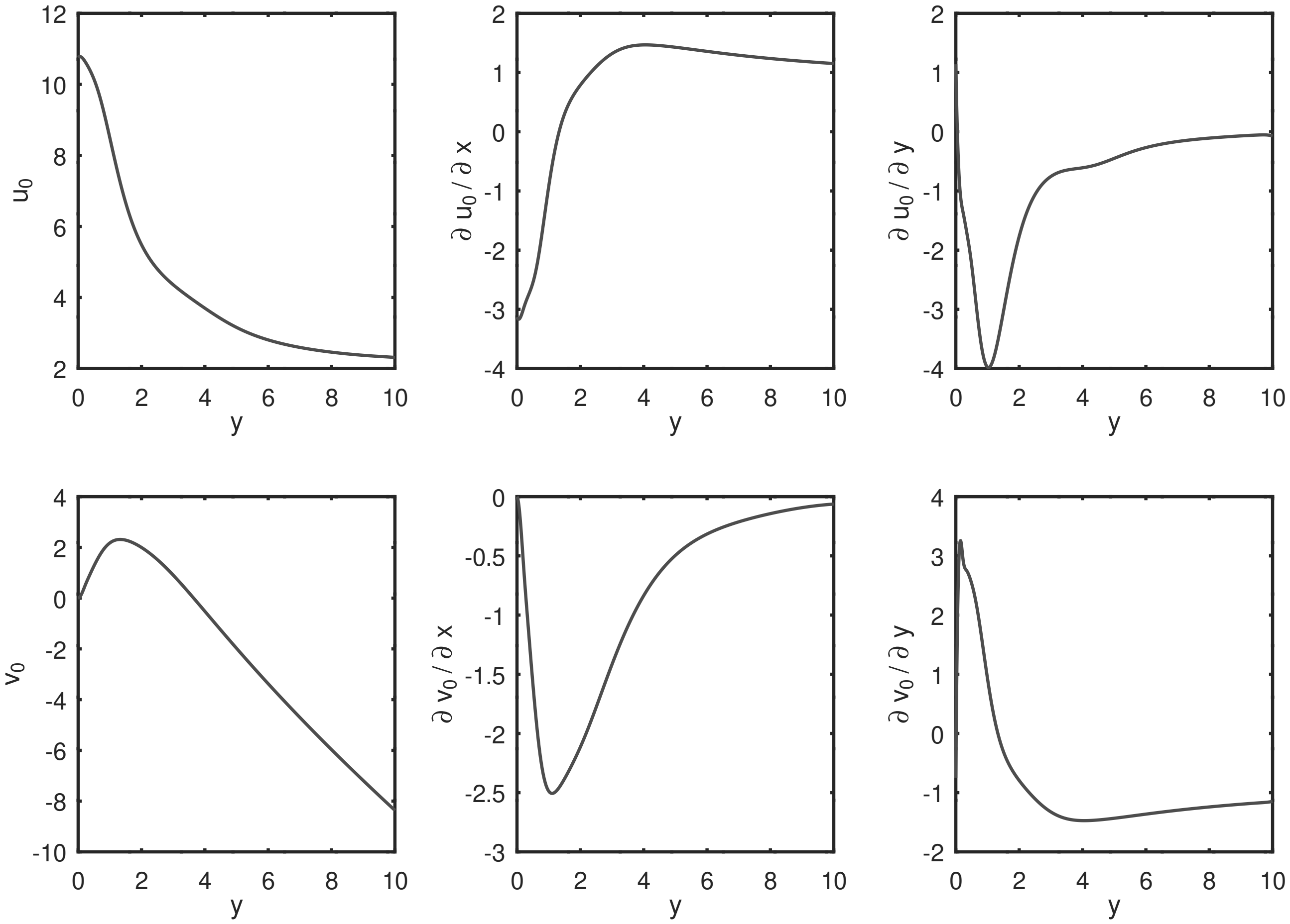


(a)

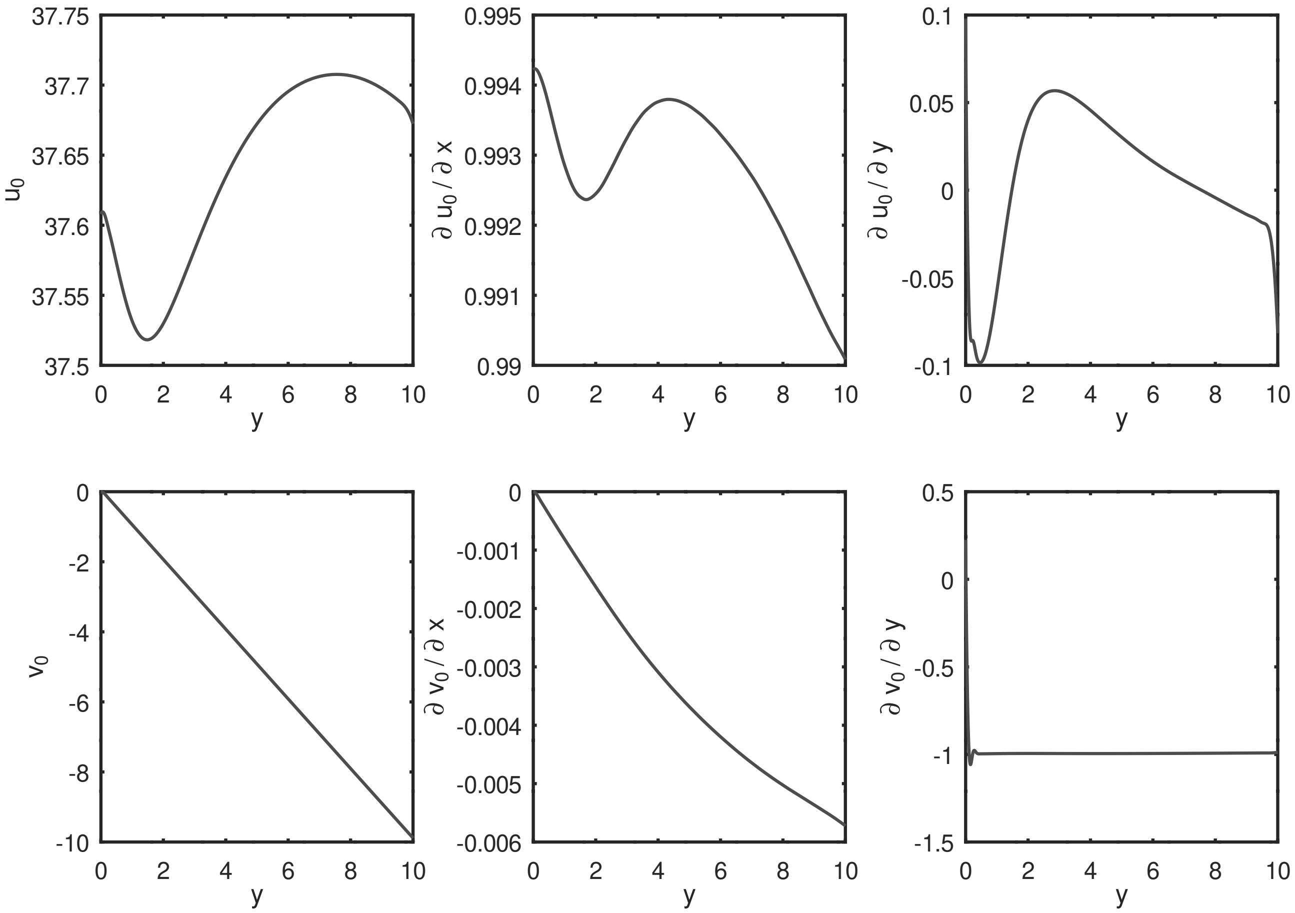


(b)

Figure 4: Counterflow (a) and coflow (b) profiles for the case $(S, Pe_b, Pe_c) = (1, 17, 17)$; (a) $x = 2$ and (b) $x = 9.31$.



(a)



(b)

Figure 5: Counterflow (a) and coflow (b) profiles for the case $(S, Pe_b, Pe_c) = (1, 17, 1)$; (a) $x = 2$ and (b) $x = 37.09$.

5. ACKNOWLEDGEMENTS

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