XXVIII CREEM
Congresso Nacional de Estudantes de Engenharia Mecânica
Santa Maria - RS

ABCM
Associação Brasileira de Engenharia e Ciências Mecânicas

XXVIII Congresso Nacional de Estudantes de Engenharia Mecânica
09 a 13 de maio de 2022, Santa Maria, Rio Grande do Sul, Brasil

FATIGUE ANALYSIS OF A CLAMPED STRUCTURE

José Pereira, josep9957@gmail.com[1]
Simone S. Hoefel, simone.santos@ufpi.edu.br[1]
René Q. Rodriguez, rene.rodriguez@ufsm.br[2]

[1]Dept. of Mechanical Engineering, Federal University of Piauí, Teresina, 64049-550, Piauí, Brazil,
[2]Dept. of Mechanical Engineering, Federal University of Santa Maria, Santa Maria, 97105-900, Rio Grande do Sul, Brazil.

Abstract. *In engineering projects, specifically in the mechanical area, it is indispensable to consider how the loads act on the components, because they can present themselves in different ways and cause abrupt failures. Therefore, the study that promotes the development of safe structures and with a longer useful life, is of significant importance. In this work, using the modified Goodman criterion, fatigue damage is evaluated in a cantilever beam with a hole in the domain subjected to different load conditions. The formulation of the finite element method and the mentioned failure criterion are presented. The stress field is obtained using a linear quadrilateral element, with four nodes per element and two degrees of freedom per node. The results obtained in computer programming, show the effect of fatigue in the structure for three different load conditions, it is evidenced greater resistance to failure in application of distributed load and lesser resistance under oblique point load.*

Keywords: *Finite element method, Fatigue failure, Stress, Load conditions*

1. INTRODUCTION

According to Callister and Rethwisch (2018), fatigue is a form of failure that occurs in structures subjected to dynamic and fluctuating stresses. The fatigue studies on structures have been carried out since the 19th century in the West, after several disastrous railway accidents, but only in the 1990s did the subject gain greater attention. This is mainly due to the growth in the use of critical components in industries such as automotive, high-speed rail and aerospace (Smith and Hillmansen, 2004). Over the last decades, several numerical modeling techniques have been used to analyze the behavior of complex structures, such as the Finite Element Method (FEM). Furthermore, the S-N curve of the structure or its critical details can be estimated from basic information about the fatigue strength of the material, reducing the time and cost of experimental tests.

Deng *et al.* (2021) applied the FEM to fatigue failure studies of Cement Asphalt mortar (AC mortar) on train tracks. In this work, the high cycle fatigue damage constitutive relationship of the mortar is developed as a material subroutine that incorporates the effects of several key factors in the finite element model: such as voids, initial deterioration and the effect of load of the wheel in the accumulation of fatigue damage in the AC. Pathak *et al.* (2014) investigated the fatigue of homogeneous and bimaterial plates with interfacial cracks under mechanical and thermal loads based on the Galerkin method. Kumar *et al.* (2015) modeled the propagation of cracks in a single element of isotropic and bimaterial materials in the presence of inclusions and voids. Bhardwaj *et al.* (2015, 2016) analyzed the interfacial crack problem of heterogeneous materials and materials with two-layer functional classification based on extended isogeometric analysis.

In more recent works, researchers began to employ topology optimization techniques to improve the performance of structures subject to fatigue. Nabaki *et al.* (2018) applied the Bi-directional Evolutionary Structural Optimization (BESO) method to minimize fatigue-restricted structural volumes. In addition to the highly nonlinear behavior of stress with respect to design variables, static and dynamic failure problems include stress singularities and many constraints. Therefore, the application of fatigue constraints in topology optimization is considered a complex engineering problem due to the nonlinear nature of the constraints. In this article, the FEM will be used to analyze the fatigue damage of continuous structures under different loading conditions based on the modified Goodman criterion.

2. METHODOLOGY

2.1 Finite Elements - Quad4 Linear Element

The finite element method is the discretization numerical technique adopted in this work, using the linear elastic quadrilateral plane isoparametric finite element with four nodes and two degrees of freedom (displacements in the vertical and horizontal directions) per node. Each element of the system is associated with a stiffness matrix, called an elementary matrix given by (Petyt, 2010):

$$\mathbf{K}^{(\mathbf{e})} = \int_{-1}^{+1}\int_{-1}^{+1} h\mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}\,\det[\mathbf{J}]\mathrm{d}\xi\mathrm{d}\eta, \tag{1}$$

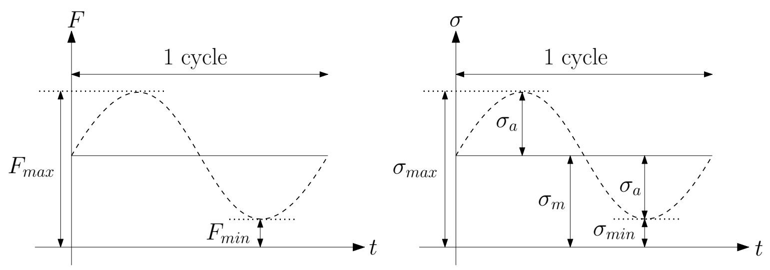
where h is the out-of-plane thickness, $\mathbf{B}$, $\mathbf{D}$ and $\mathbf{J}$ are the strain, elasticity and Jacobian matrices, respectively. Furthermore, ξ and η are defined as the isoparametric coordinates. In the treatment of FEM, the equilibrium equation for a structure under static load is given by:

$$\mathbf{K}\mathbf{U} = \mathbf{F}, \tag{2}$$

being $\mathbf{K}$ the global stiffness matrix of the structure, $\mathbf{U}$ the vector of nodal displacements and $\mathbf{F}$ the vector of nodal forces.

2.2 Fatigue Theory

The fatigue degradation process is associated with deterioration under cyclic loading. To assess the damage, the High Cycle Fatigue (HCF) regime is employed with constant amplitude proportional sinusoidal loading conditions, as shown in Fig. 1a, where $F_{\max}$ and $F_{\min}$ are the maximum and minimum applied load, respectively. The stress state history shown in Fig. 1b is obtained by applying a sinusoidal load to the structure and then calculating the stress amplitude (σ_a) and the mean stress (σ_m) from the maximum stress value (σ_{max}) and the minimum stress value (σ_{min}).



(a) Sinusoidal load (b) One cycle stress history.

Figure 1: One cycle of the stress history in HCF.
Source: Adapted from Nabaki *et al.* (2019).

As a way of reducing the complexity of the dynamic evaluation, the adaptations proposed by the equivalent static analysis are adopted, where $F = F_{\max}$ is statically applied to obtain the stress distribution. In this sense, the stress vectors, $\boldsymbol{\sigma}_i$, are calculated at the center of the element as follows:

$$\boldsymbol{\sigma}_i = \mathbf{D}_i\mathbf{B}_i\mathbf{u}_i, \tag{3}$$

where, $\mathbf{u}_i$ is the vector of nodal displacements for element i. The alternating and mean stresses are calculated from the stress values expressed as:

$$\boldsymbol{\sigma}_{a_i} = c_a\boldsymbol{\sigma}_i = \begin{bmatrix} \sigma_{x_{a_i}} \\ \sigma_{y_{a_i}} \\ \tau_{xy_{a_i}} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\sigma}_{m_i} = c_m\boldsymbol{\sigma}_i = \begin{bmatrix} \sigma_{x_{m_i}} \\ \sigma_{y_{m_i}} \\ \tau_{xy_{m_i}} \end{bmatrix}, \tag{4}$$

where, c_a and c_m are amplitude and mean stress scaling factors, respectively, given by:

$$c_a = \frac{1-(F_{\min}/F_{\max})}{2} \quad \text{and} \quad c_m = \frac{1+(F_{\min}/F_{\max})}{2}. \tag{5}$$

The von-Mises stress calculation is used to calculate the mean and alternating stresses of the elements as follows:

$$\begin{aligned} \sigma_{a_i}^{\text{vonMises}} &= \sqrt{\sigma_{x_{a_i}}^2 + \sigma_{y_{a_i}}^2 - \sigma_{x_{a_i}}\sigma_{y_{a_i}} + \tau_{xy_{a_i}}^2}, \\ \sigma_{m_{\mathrm{i}}}^{\text{vonMises}} &= \sqrt{\sigma_{x_{m_i}}^2 + \sigma_{y_{m_i}}^2 - \sigma_{x_{m_i}}\sigma_{y_{m_i}} + \tau_{xy_{m_i}}^2}. \end{aligned} \tag{6}$$

The evaluation of fatigue failure is realized by applying the modified Goodman criterion expressed by:

$$L_i^{GM}(x) = \frac{\sigma_{a_i}^{\text{vonMises}}}{(\sigma_i)_{N_f}} + \frac{\sigma_{m_i}^{\text{vonMises}}}{\sigma_{ut}} \leq 1. \tag{7}$$

For the application of principal stresses, the criterion is represented in another way, as revealed in Fig. 2, where the alternating stress is limited by the critical fatigue stress ($(\sigma_i)_{N_f}$), considering an infinite number of life cycles ($N_f > 10^7$). To avoid fatigue failure, all combinations of alternating and average stresses for all elements of the structure must be within the gray area of the diagram (Fig 2), where σ_{ut} and σ_y represent ultimate and yielding stress, respectively.

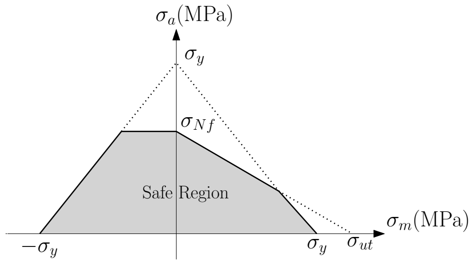


Figure 2: Modified Goodman Diagram.
Source: Adapted from Nabaki *et al.* (2019).

The value of the $(\sigma_i)_{N_f}$ alternating stress is obtained by the Basquin equation (Nabaki *et al.*, 2019):

$$(\sigma_i)_{N_f} = \sigma'_f \left(2N_f\right)^b , \tag{8}$$

in this expression σ'_f and b are the coefficient and exponent of fatigue strength, respectively.

To verify the structure's resistance to failure, three different combinations of mean and alternating principal stresses are considered, where the results of these combinations in each element are placed in the diagram. The three combinations are written as follows (Nabaki *et al.*, 2019):

Combination 1:

$$\begin{aligned} &\sigma_a = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a}} \qquad \text{and} \\ &\sigma_m = \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}} \,. \end{aligned} \tag{9}$$

Combination 2:

$$\begin{aligned} &\sigma_a = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a}} \qquad \text{and} \\ &\sigma_m^{eq} = \sigma_{1m} + \sigma_{2m} \,. \end{aligned} \tag{10}$$

Combination 3:

$$\begin{aligned} &\sigma_a = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a}} \qquad \text{and} \\ &\sigma_m^{\text{signed vonMises}} = \begin{cases} \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}} & \text{if } |\sigma_{1m}| \geq 0 \\ -\sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}} & \text{if } |\sigma_{1m}| < 0 \end{cases} . \end{aligned} \tag{11}$$

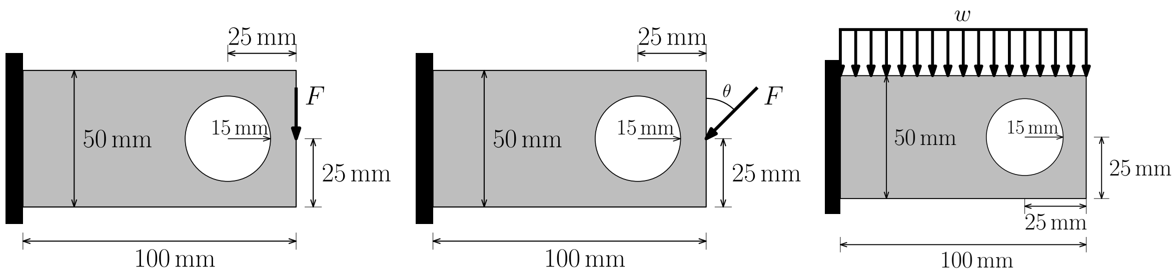
In this approach, the mean stress can also assume negative values, representing the material under compressive cyclic loading conditions, a generic characteristic of most dynamic loads.

2.3 Computational Implementation

Based on the finite element formulation, in a MATLAB environment, presented by Picelli *et al.* (2020), available at the link (here), the routine for calculating stresses and the modified Goodman criteria are implemented. After inserting the geometric parameters, material and load and boundary conditions to the pre-processing of the aforementioned computational programming, the fatigue results of the structural problem are obtained.

3. RESULTS

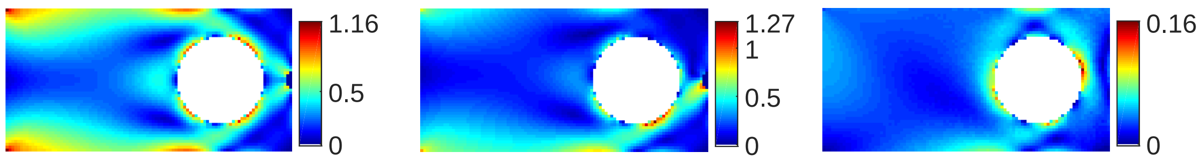
The computer code validation step is presented in a previous work entitled Fatigue Analysis Using The Finite Element Method (Pereira *et al.*, 2021) by the present authors. For current analysis, a cantilever beam containing a hole in the domain is considered, the dimensions, boundary conditions and load are shown in Fig. 3. The out-of-plane thickness, modulus of elasticity and Poisson's ratio are defined as $1\mathrm{mm}$, $210\,\mathrm{GPa}$ and 0.3, respectively. The load is applied with $F_{max} = 300\,\mathrm{N}$ and $F_{min} = 800\,\mathrm{N}$ in three different ways, first as a vertical point load (Fig. 3a), second as an inclined point load at $\theta = 45°$ (Fig. 3b) and lastly as a distributed load $w = 8\,\mathrm{N/m}$ (Fig. 3c). To construct the failure criteria diagrams, the parameters $\sigma'_f = 493\,\mathrm{MPa}$, $\sigma_{ut} = \sigma_y = 358\,\mathrm{MPa}$, $N_f = 10^7$ and $b = -0.086$ are used. Furthermore, a discretization with Quad4 elements ($1\,\mathrm{mm}$ x $1\,\mathrm{mm}$) is used.



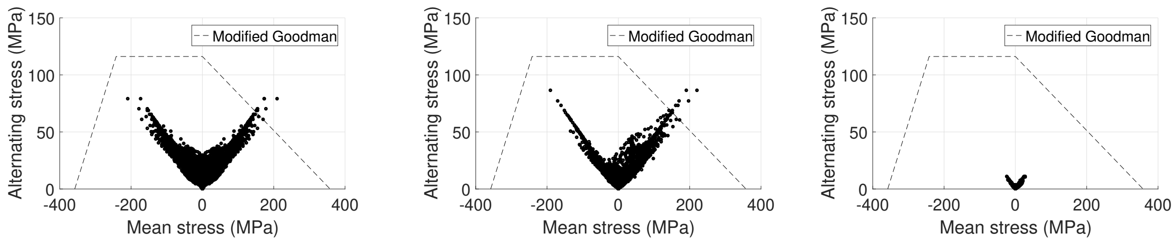
(a) Transverse load. (b) Inclined load. (c) Distributed load.

Figure 3: Beam clamped under three loading conditions.

Therewith, the results shown in Fig. 4 are obtained. In these graphs, the effects of stress concentration around the applied point load are disregarded, ten elements in the proximities of the force are excluded from the failure criteria. The colors in the geometry domain (Fig. 4a, 4b and 4c) represent the fatigue damage in the structure, thus highlighting the critical regions of the structure, L_i^{GM} values greater than one indicate fatigue failure. The Figures 4d, 4e and 4f present the modified Goodman diagram, which considers the fatigue analysis from the mean and alternating principal stresses. In these figures (4d, 4e and 4f), points outside the safe zone of the diagram, that is, outside the region delimited by the dashed line, indicate fatigue failure.



(a) L_i^{GM} for transverse load. (b) L_i^{GM} for inclined load. (c) L_i^{GM} for distributed load.



(d) Diagram - transverse load. (e) Diagram - inclined load. (f) Diagram - distributed load.

Figure 4: Fatigue in a cantilever beam.

The results in Fig. 4a and 4d refer to the application of the vertical point load. It can be seen from the color map that the structure is more affected in the regions of the vertices connected to the clamp. Figures 4b and 4e present the responses due to the application of the inclined point load, in the color distributions, more expressive damage is observed in a region tangent to the hole. In the responses considering the load as distributed are shown in Fig. 4c and 4f, greater requests can be observed in regions located around the hole. The case with highest L_i^{GM} values among the three cases, the acting of the inclined load, causes more expressive damages.

As demonstrate the Fig. 4a and 4b, in the two cases there is fatigue failure since they present L_i^{GM} values greater than one, the same conclusion can be made when evaluating Fig. 4d and 4e, they show in the modified Goodman diagram points outside the safe zone. On the other hand, Fig. 4c and 4f show that by distributing the loading along the entire length of the structure, fatigue failure does not occur. In view of the foregoing, it is clear that fatigue failure can be significantly affected with alterations in load application conditions.

4. CONCLUSIONS

Fatigue damage in a cantilever structure subjected to different load conditions was evaluated using the modified Goodman failure criterion with the application of FEM. The acting of a certain loading was considered in three ways: vertical punctual, oblique punctual and distributed. In the first, the structure is more affected in the regions of the vertices connected to the crimp, in the second, there is a certain region tangent to the most required hole, in both load applications fatigue failure was verified, with more expressive damage under inclined load. Lastly, in the case of distributed loads, there was no failure, the greatest solicitations are located in regions around the hole. Therefore, it is noted that fatigue damage can be significantly affected with alterations in the conditions of acting of external loads.

5. ACKNOWLEDGEMENTS

The authors thank Brazilian Association of Engineering and Mechanical Sciences ABCM (*Associação Brasileira de Engenharia e Ciências Mecânicas*) for financial support through the 2021 ABCM Scientific Initiation Scholarship Notice.

6. REFERENCES

Bhardwaj, G., Singh, I. and Mishra, B., 2015. "Stochastic fatigue crack growth simulation of interfacial crack in bi-layered fgms using xiga". *Computer Methods in Applied Mechanics and Engineering*, Vol. 284, pp. 186–229.

Bhardwaj, G., Singh, S., Singh, I., Mishra, B. and Rabczuk, T., 2016. "Fatigue crack growth analysis of an interfacial crack in heterogeneous materials using homogenized xiga". *Theoretical and Applied Fracture Mechanics*, Vol. 85, pp. 294–319.

Callister, W.D. and Rethwisch, D.G., 2018. *Materials science and engineering: an introduction*, Vol. 9. Wiley New York.

Deng, S., Ren, J., Wei, K., Ye, W., Du, W. and Zhang, K., 2021. "Fatigue damage evolution analysis of the ca mortar of ballastless tracks via damage mechanics-finite element full-couple method". *Construction and Building Materials*, Vol. 295, p. 123679.

Kumar, S., Singh, I., Mishra, B. and Rabczuk, T., 2015. "Modeling and simulation of kinked cracks by virtual node xfem". *Computer Methods in Applied Mechanics and Engineering*, Vol. 283, pp. 1425–1466.

Nabaki, K., Shen, J. and Huang, X., 2018. "Bi-directional evolutionary topology optimization based on critical fatigue constraint". *International Journal of Civil and Environmental Engineering*, Vol. 12, No. 2, pp. 113–118.

Nabaki, K., Shen, J. and Huang, X., 2019. "Evolutionary topology optimization of continuum structures considering fatigue failure". *Materials & Design*, Vol. 166, p. 107586.

Pathak, H., Singh, A. and Singh, I.V., 2014. "Fatigue crack growth simulations of homogeneous and bi-material interfacial cracks using element free galerkin method". *Applied Mathematical Modelling*, Vol. 38, No. 13, pp. 3093–3123.

Pereira, J., Hoefel, S.S. and Rodriguez, R.Q., 2021. "Fatigue analysis using the finite element method". In *XLII Ibero-Latin-American Congress on Computational Methods in Engineering*. Natal, Rio Grande do Norte, Brazil.

Petyt, M., 2010. *Introduction to finite element vibration analysis*. Cambridge university press.

Picelli, R., Ranjbarzadeh, S., Sivapuram, R., Gioria, R. and Silva, E., 2020. "Topology optimization of binary structures under design-dependent fluid-structure interaction loads". *Structural and Multidisciplinary Optimization*, Vol. 62, No. 4, pp. 2101–2116.

Smith, R. and Hillmansen, S., 2004. "A brief historical overview of the fatigue of railway axles". *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, Vol. 218, No. 4, pp. 267–277.

7. RESPONSIBILITY FOR INFORMATION

The authors confirm that they are solely responsible for the authorship of this work and that all material included herein as part of this work is the property (and authorship) of the authors, or has the owners' permission to be included here.