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EPTT 0114: TURBULENT FORCED CONVECTION IN THE THERMAL ENTRANCE OF RECTANGULAR DUCTS: ANALYSIS FOR DIFFERENT MODELS OF VELOCITY DISTRIBUTION AND MOMENTUM EDDY DIFFUSIVITY

D. L. A. VELOSO [1]
C. A. C. DOS SANTOS[2]
[1] Instituto Federal da Paraíba, Jaguaribe, João Pessoa - PB - Brasil - CEP:58015-430
[2] Universidade Federal da Paraíba, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
dhiego_vel@hotmail.com, carloscabralsantos@yahoo.com.br
F. A. LIMA [3]
[3] Instituto Federal da Paraíba, Jd Oásis, Cajazeiras - PB - Brasil - CEP:58900-000
fabioarsp@gmail.com

Resumo: *This work describes a hybrid solution (analytical-numerical) for the heat transfer by forced convection in the entry section of a rectangular duct, considering the turbulent flow of dynamically developed and thermally developing Newtonian fluids. The energy equation is solved using the Classical Integral Transform Technique (CITT), while the associated eigenvalue problem is solved using the Generalized Integral Transform Technique (GITT). Different turbulence models with different formulations for velocity distribution and momentum eddy diffusivity are considered. The temperature field and the local Nusselt number are evaluated for various values of the Reynolds and Prandtl numbers. For each turbulence model considered, the thermal input length is determined. The results, presented in the form of tables and graphs, are compared with reference values in the specialized literature.*

Keywords:*Turbulent Flow, CITT, GITT, Temperature Field, Nusselt Number.*

1. INTRODUCTION

The study of turbulence is of great importance in engineering, given the large number of practical applications in which it is present. In heat transfer, the involvement of fluid in turbulent motion appears in most processes involving the transport of energy. Over the years, many theories and concepts have been formulated in an attempt to obtain a universalist description of the phenomenon of turbulence that is suitable for any problem of practical interest. However, due to the existence of irregular fluctuations, there is still neither a univocal form nor a fundamental theory for its treatment. While this general formulation is not achieved, simplified models have been proposed as a way to analyze specific problems in each area of interest. The methodology used consists of observing patterns of behavior in each situation analyzed. These patterns establish the so-called turbulence models, which are based on empirical or semi-empirical relationships and make this study viable. In the case of the distribution of velocities in a turbulent flow, for example, Nikuradse(1950) was the initial investigator and presented careful measurements considering the flow along rough tubes; since then, several other researchers have devoted attention and made new attempts in order to develop empirical relationships that are appropriate, with the greatest possible precision, and that allow establishing a universal speed profile.

The scope of the work is focused on the analysis of heat transfer in turbulent forced convection, which has received wide attention from the community due to the constant search for calculations of parameters of practical interest that allow to develop projects and build optimized thermal devices. However, the calculations of these parameters, such as friction factors and heat transfer coefficients, are totally dependent on the turbulence model adopted to describe the speed distribution and the momentum eddy diffusivity, and consequently different results are found for each situation considered, (Santos et al., 2001).

The turbulent forced convection in the thermal entrance region of rectangular ducts has been extensively studied, considering several models of turbulence and several boundary conditions. In the works of (Sleicher et al., 1970; Notter and Sleicher, 1971; Notter and Sleicher, 1972; Shibani and Özisik, 1977; Özisik et al., 1989; Santos et al., 1995; Brown et al., 1997; Quaresma et al., 2001), it is possible to make a comprehensive review of the turbulent forced convection in thermal developing and hydrodynamically developed flow, object of study of the present work.

The objective of this work is to analyze the thermal development in the turbulent flow of Newtonian fluids, considering different turbulence models. Four turbulence models are considered as references, based on the work of

(Prandtl, 1910 and Taylor, 1916; Von Karman, 1939; Rannie, 1956; Reichardt, 1951 and Spalding, 1961). The combination of the classical integral transform technique (CITT) and the generalized integral transform technique (GITT) is used to solve the equations of the proposed problem; CITT is applied to the main equation and GITT is applied to the associated eigenvalue problem. Analytical solutions for the thermal development of turbulent flow in ducts present difficulties associated with the calculation of eigenvalues and eigenfunctions of the related eigenvalue problem, as described by Santos et al., 2001, Özisik et al., 1989. These adversities are overcome using the renowned generalized integral transformation technique (GITT). The GITT is used to solve the Sturm-Liouville problem associated with the original problem, as described by Cotta, 1993. The application of GITT to Sturm-Liouville problems is capable of transforming the differential eigenvalue problem into an algebraic eigenvalue problem, which is easier to solve; in parallel to this, GITT is a technique that can manage irregular domains with some ease, overcoming the mathematical difficulties imposed by the turbulent models hydrodynamically developed in layers.

This article is organized into five sections. In section 2, the mathematical modeling of the proposed physical problem is carried out. In Section 3, the application of CITT in the governing equation is discussed and the solution of the thermal field and parameters of practical interest is presented. In Section 4, the results obtained in the present work are shown and discussed. In Section 5, final considerations are made.

2. MATHEMATICAL MODELING

The following simplifying assumptions are considered in the problem analysis:

- Steady forced convection in thermally developing, hydrodynamically developed flow;
- Viscous dissipation, free convection and axial conduction effects are neglected;
- Physical properties are taken as constant;
- The duct wall is subjected to a uniform temperature (Tw);
- The fluid enters the duct with a constant temperature (Ti).

The mathematical formulation for this forced convection problem is dimensionless form is written as:

Energy equation

$$U(R)\frac{\partial\Theta(X,R)}{\partial X}=\frac{1}{R^m}\frac{\partial}{\partial R}\left[R^m E_h(R)\frac{\partial\Theta(X,R)}{\partial R}\right];\ X>0\ and\ \ 0<R<1 \tag{1}$$

where, the constant m is related to the geometry of the duct. If m=0 the duct is rectangular (parallel-plates channel).

Boundary Conditions

$$\frac{\partial\Theta(X,R)}{\partial R}=0;\ X>0\ and\ \ R=0 \tag{2}$$

$$\Theta(X,R)=0;\ X>0\ and\ \ R=1 \tag{3}$$

Inlet condition

$$\Theta(X,R)=1;\ X=0\ and\ \ 0\le R\le 1 \tag{4}$$

For the analysis of the problem the following dimensionless parameterswere defined, given by equations (5a-i), with the objective of solving not only a particular problem, but a class of problems that are defined by the same proposed model. Where: ε_m -dynamic turbulent diffusivity and ε_h - turbulent thermal diffusivity.

$$X=\frac{2^{(4-2m)}\left(x/D_h\right)}{C.\mathrm{Re}.\mathrm{Pr}}\ ;\ R=\frac{r}{r_0}\ ;\ U(R)=\frac{u(r)}{u_{\max}}=\frac{u(r)}{Cu_m}\ ;\ C=\frac{u_{\max}}{u_m}\ ;\ \mathrm{Pr}=\frac{\nu}{\alpha} \tag{5a-e}$$

$$Pr_t = \frac{\varepsilon_m}{\varepsilon_h} \; ; \; Re = \frac{D_h . u_m}{\nu} \; ; \; \Theta(X,R) = \frac{T(x,r) - T_w}{T_i - T_w} \; ; \; E_h(R) = 1 + \frac{\varepsilon_h}{\alpha} = 1 + \frac{Pr}{Pr_t}\frac{\varepsilon_m}{\nu} \tag{5f-i}$$

where ν is the kinematic viscosity, α is the thermal diffusivity of the fluid, r0 is the characteristic length, Dh= 2(2-m).r0 is the hydraulic diameter and Pr, Re and Prt are, respectively, the numbers of Prandtl, Reynolds and turbulent Prandtl.

In this paper adopted different turbulence models will be adopted, with different formulations for velocity distribution and for the momentum eddy diffusivity. Four situations will be analyzed, based on analytical expressions proposed for the universal velocity profile - Law of the Wall (Kestin and Richardson,1963 ; Kakaç et. al, 2014 and Santos et al.,2001).

Case 1: The turbulence model is the fully-developed two-layer model for velocity distribution together with two-layer model for the momentum eddy diffusivity, based on the works of Prandtl(1910), Taylor (1916)and Schlichting (1960).

The two-layer turbulent velocity distribution is taken as:

$$u^+ = y^+ ; \quad for \quad 0 \le y^+ \le 11.5 \quad \text{, laminar sublayer} \tag{6}$$

$$u^+ = 5.5 + 2.5 \ln(y^+) ; \quad for \quad y^+ > 11.5 \quad \text{, turbulent core} \tag{7}$$

The two-layer model for the momentum eddy diffusivity is taken as:

$$E_h(R) = 1 + \frac{Pr}{Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 \; ; \qquad for \quad 0 \le y^+ \le 11.5 \tag{8}$$

$$E_h(R) = 1 + \frac{Pr}{Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{Pr}{Pr_t}\left(0.4 y^+ - 1\right); \qquad for \quad y^+ > 11.5 \tag{9}$$

Case 2: The turbulence model is the fully-developed three-layer model for velocity distribution together with three - layer model for the momentum eddy diffusivity, based on the works of Von Karman (1939).

The three-layer turbulent velocity distribution is taken as:

$$u^+ = y^+ ; \quad for \; 0 \le y^+ < 5 \quad \text{, laminar sublayer} \tag{10}$$

$$u^+ = -3.05 + 5 \ln(y^+) ; \quad for \; 5 \le y^+ \le 30 \quad \text{, buffer layer} \tag{11}$$

$$u^+ = 5.5 + 2.5 \ln(y^+) ; \quad for \quad y^+ > 30 \quad \text{, turbulent core} \tag{12}$$

The three-layer model for the momentum eddy diffusivity is taken as:

$$E_h(R) = 1 + \frac{Pr}{Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 \; ; \qquad for \; 0 \le y^+ < 5 \tag{13}$$

$$E_h(R) = 1 + \frac{Pr}{Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{Pr}{Pr_t}\left(0.2 y^+ - 1\right); \qquad for \; 5 \le y^+ \le 30 \tag{14}$$

$$E_h(R) = 1 + \frac{Pr}{Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{Pr}{Pr_t}\left(0.4 y^+ - 1\right); \qquad for \quad y^+ > 30 \tag{15}$$

Case 3: The turbulence model is the fully-developed two-layer model for velocity distribution together with two-layer model for the momentum eddy diffusivity, based on the works of Rannie (1956).

The two-layer turbulent velocity distribution is taken as:

$$u^{+} = 14.53 \tanh\left(\frac{y^{+}}{14.53}\right) \;; \qquad for \quad 0 \le y^{+} \le 27.5 \tag{16}$$

$$u^{+} = 5.5 + 2.5 \ln\left(y^{+}\right) \;; \qquad for \;\; y^{+} > 27.5 \tag{17}$$

The two-layer model for the momentum eddy diffusivity is taken as:

$$E_h(R) = 1 + \frac{\Pr}{\Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{\Pr}{\Pr_t}\left[\sinh^2\left(\frac{y^{+}}{14.53}\right)\right] \;; \qquad for \quad 0 \le y^{+} \le 27.5 \tag{18}$$

$$E_h(R) = 1 + \frac{\Pr}{\Pr_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{\Pr}{\Pr_t}\left(0.4y^{+} - 1\right); \qquad for \;\; y^{+} > 27.5 \tag{19}$$

Case 4: The turbulence model is the fully-developed three-layer model for velocity distribution (Reichardt,1951; Kays and Crawford, 1980), together with two-layer model for the momentum eddy diffusivity (Reichardt,1951 and Spalding,1961).

The three-layer turbulent velocity distribution is taken as:

$$u^{+} = y^{+} \;; \quad for \;\; 0 \le y^{+} < 5 \quad \text{, laminar sublayer} \tag{20}$$

$$u^{+} = -3.05 + 5 \ln(y^{+}) \;; \quad for \;\; 5 \le y^{+} \le 30 \quad \text{, buffer layer} \tag{21}$$

$$u^{+} = 5.5 + 2.5 \ln\left[y^{+} \frac{1.5(1+R)}{\left(1+2R^2\right)}\right] \;; \quad for \;\; y^{+} > 30 \quad \text{, turbulent core} \tag{22}$$

The two-layer model for the momentum eddy diffusivity is taken as:

$$\frac{\varepsilon_m}{\nu} = \frac{k_2}{E}\left[e^{k_2 u^{+}} - 1 - k_2 u^{+} - \frac{\left(k_2 u^{+}\right)^2}{2!} - \frac{\left(k_2 u^{+}\right)^3}{3!}\right] \;; \quad for \;\; y^{+} < 40 \tag{23}$$

$$\frac{\varepsilon_m}{\nu} = \frac{k_1 R^{+}}{6}\left(1-R^2\right)\left(1+2R^2\right) \;; \quad for \;\; y^{+} \ge 40 \tag{24}$$

where k_1=0.4, k_2=0.407 and E=10.

Several dimensionless groups were included in equations (6-24), which are defined in the following form:

$$y^{+} = (1-R)R^{+} \;; \qquad R^{+} = \frac{\mathrm{Re}}{2^{(2-m)}}\sqrt{\frac{f}{8}} \;; \qquad u^{+} = \frac{u(r)}{u_m\sqrt{\frac{f}{8}}} \;; \qquad f = \frac{4\tau_w}{\frac{\rho u_m{}^2}{2}} \tag{25 a-d}$$

where f is the friction factor. In the present work, empirical correlations are used for the friction factor, based on the works of Filonenko(1954), Dean (1978) and Bhatti and Shah (1987).

Filonenko correlation for parallel-plates channel:

$$\frac{f}{4} = \left[1.58 \ln(\mathrm{Re}) - 3.28\right]^{-2} \;; \quad 1\times10^{4} \le \mathrm{Re} \le 1\times10^{7} \tag{26}$$

Dean correlation for parallel-plates channel:

$$\frac{f}{4}=0.073\left(\frac{\mathrm{Re}}{2}\right)^{-0.25} \quad ; \quad 1.2\times10^{4}\leq \mathrm{Re}\leq 1.2\times10^{6} \tag{27}$$

Bhatti and Shah correlation for parallel-plates channel:

$$\frac{f}{4}=0.00128+\frac{0.1143}{\mathrm{Re}^{1/3.2154}} \quad ; \quad 4\times10^{3}\leq \mathrm{Re}\leq 1\times10^{7} \tag{28}$$

3. APPLICATION OF THE CLASSICAL INTEGRAL TRANSFORMING TECHNIQUE

The CITT presents itself as an established methodology, having been used successfully in several classes of heat transfer models and fluid mechanics, as can be seen in Mikhailov (1984). In order to apply the technique, it is necessary to establish an auxiliary eigenvalue problem, as well as to define a transform-inverse pair in order to reduce the original problem, which is a partial differential equation, in a system of ordinary differential equations. Secondly, the inverse formula can be used to obtain the solution to the original problem proposed by Cotta (1993), (1998).

3.1 Auxiliary problem of eigenvalue

The auxiliary problem for determining the temperature field is taken as:

$$\frac{1}{R^m}\frac{\partial}{\partial R}\left(R^m E_h(R)\frac{\partial \Psi_i(\mu_i,R)}{\partial R}\right)+\mu_i^2 U(R)\Psi_i(\mu_i,R)=0,\ 0<R<1 \tag{29}$$

$$\frac{\partial \Psi_i(\mu_i,R)}{\partial R}=0;\ R=0 \text{ e } \mu_i>0 \tag{30}$$

$$\Psi_i(\mu_i,R)=0;\quad R=1 \text{ e } \mu_i>0 \tag{31}$$

Several studies point out great difficulties in the calculation of eigenvalues in the mentioned auxiliary problem, due to the limitations in the numerical schemes used by Santos et al. (2001) and Özisik et al. (1989). This fact delayed the emergence of analytical solutions for the thermal field in the study of turbulent forced convection. Özisik et al. (1989), Brown et al. (1997) and Santos et al. (1995) used the signal counting method of Mikhailov (1983), (1984), to circumvent the difficulties associated with the eigenvalue problem. Cotta (1993) developed a method using the integral transformation technique, based on the ideas of the generalized integral transform technique (GITT), which allows solving eigenvalue problems with a high degree of difficulty. In the present work, GITT is used to determine the eigenvalues (μi), the eigenfunctions, Ψi (ς), and the norms (Ni), as described by Cotta (1993). The generalized integral transformation method was implemented in computational code with the aid of the Wolfram Mathematica 12.0 software, to solve the associated eigenvalue problem.

3.2 Integral transformation of the temperature field

The pair transformed integral, defined for this problem is given by:

$$\overline{\Theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 R^m U(R)\Psi_i(\mu_i,R)\Theta(X,R)dR, \qquad \textit{Transform} \tag{32}$$

$$\Theta(X,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(X)}{N_i^{1/2}} \qquad \textit{Inverse} \tag{33}$$

Applying integral operators in equation (1), with the aid of the auxiliary problem and the transformed-inverse pair, it is possible to transform this partial differential equation into a system of ordinary differential equations given by:

$$\frac{d\overline{\Theta}_i(X)}{dX} = -\mu_i^2.\overline{\Theta}_i(X) \tag{34}$$

This system has a classical analytical solution, given by:

$$\overline{\Theta}_i(X) = \overline{\Theta}_i(0).e^{-\mu_i^2 X} \tag{35}$$

Where:

$$\overline{\Theta}_i(0) = \frac{1}{N_i^{1/2}}\int_0^1 R^m U(R).\Psi_i(\mu_i, R)dR = \bar{f}_i \tag{36}$$

At this point, it is worth noting that the application of CITT in solving the proposed problem is equivalent to the application of the traditional method of separation of variables. As CITT is a generalization of the method of separation of variables, making it possible to extend the analysis to cases in which the source terms are not null, there is no burden on the choice made in the present study. On the contrary, the model discussed in the present work can be taken as a reference and can later be extended to more generic models that impose restrictions on the use of the variable separation method.

3.3 Temperature field solution

Using the inverse formula it is possible find the general solution of the temperature field for the proposed physical problem. The temperature field for the thermal input region takes the form:

$$\Theta(X, R) = \sum_{i=1}^{\infty} \frac{\Psi_i(\mu_i, R)\bar{f}_i.e^{-\mu_i^2 X}}{N_i^{1/2}} \tag{37}$$

From this solution it is possible to calculate the average temperature and the number of local Nusselt through the expressions:

$$\Theta(X)_{average} = \frac{\int_0^1 R^m U(R)\Theta(X, R)dR}{\int_0^1 R^m U(R)dR} \tag{38}$$

$$Nu(X) = -\frac{2.(2-m)}{\Theta(X)_{average} - \Theta(X,1)} \left.\frac{d\Theta(X,R)}{dR}\right|_{R=1} \tag{39}$$

For the evaluation of the asymptotic Nusselt number and validation of the results obtained in the present work, the expression obtained by Gnielinski (1976), is used given by:

$$Nu_\infty = \frac{\left(f/2\right)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7\left(f/2\right)^{\frac{1}{2}}\left(\mathrm{Pr}^{\frac{2}{3}}-1\right)}, \quad for \quad \begin{array}{c} 2.3\times10^3 \le \mathrm{Re} \le 5\times10^6 \\ 0.5 \le \mathrm{Pr} \le 2000 \end{array} \tag{40}$$

The expression obtained by Prandtl (1910) and Taylor (1916), is also used, given by:

$$Nu_\infty = \frac{\left(f/2\right)\mathrm{Re}\,\mathrm{Pr}}{1+5\left(f/2\right)^{\frac{1}{2}}(\mathrm{Pr}-1)} \quad , \quad for \quad \begin{array}{c} 5\times10^3 \le \mathrm{Re} \le 5\times10^6 \\ \mathrm{Pr} \le 10 \end{array} \tag{41}$$

4. RESULTS

For the purposes of benchmarking the results of the present study, results were compared to these found in the specialized literature, particularly in Gnielinski (1976), Taylor (1916), Özisik et al. (1983) and Santos et al. (2001), showing the robustness and effectiveness the combination of CITT and GITT in the solution of the proposed physical problem. In this paper the situation of a flow inside a channel of parallel flat platesis analyzed, as can be seen in presented results in tables 1 and 2 and figures (1-3), considering different values of Reynolds and Prandtl.

For all the graphs and tables contained in this section, 200 eigenvalues and 200 corresponding eigenfunctions were used, which guarantees good convergence to the solution even in the thermal entrance region. The analysis is carried out for the four established turbulence models, where it is also possible to compare the results with each other. The friction factor is computed based on empirical correlations, based on the work of Filonenko(1954), Dean (1978), Bhatti and Shah (1987). For all the cases analyzed in the present work, Prt = 1 is considered, analogously to the studies carried out by Özisik et al. (1989) and Quaresma et al. (2001). The integral balance is used to estimate the variation in the local Nusselt number until reaching the converged value.

The tables 1 and 2 show the remarkable influence that the Reynolds and Prandtl numbers exert on the prediction of the asymptotic convective Nusselt number. For a specific number of Reynolds, an increase in the number of Prandtl leads to an increase in the asymptotic Nusselt number. This behavior also occurs with the increase of Reynolds number, keeping the same number of Prandtl, as we can also observe in figure 1. The above mentioned facts make it possible to conclude that the increase in Reynolds and Prandtl numbers produces an increase in heat transfer rates, causing the axial length of thermal development to be reduced, as can be seen in Figures 2 and 3.

As predicted, the results obtained in the present study, shown in tables 1 and 2, do not show perfect compliance with the results predicted by the correlations of Gnielinski (1976) and Taylor (1916), because it is the comparison of experimental studies with analytical-numerical simulation studies. However, it is possible to observe in tables 1 and 2 a very similar variation between the results, which validates the study developed in the present work.

Table 1. Asymptotic Nusselt number considering different numbers of Reynolds and Prandtl for the turbulent flow between flat plates.

Flat Plates - Pr_t = 1								
Pr = 0.72			Pr = 1			Pr = 2		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
38.31[a]	123.91[a]	211.26[a]	41.69[a]	141.75[a]	247.16[a]	47.58[a]	177.16[a]	326.20[a]
38.17[b]	123.82[b]	210.54[b]	41.52[b]	141.64[b]	246.27[b]	47.37[b]	177.03[b]	324.86[b]
39.87[c]	129.41[c]	219.26[c]	43.46[c]	148.23[c]	257.14[c]	49.77[c]	185.74[c]	341.14[c]
36.34[d]	116.95[d]	198.48[d]	41.52[d]	138.27[d]	237.67[d]	54.51[d]	192.93[d]	339.83[d]
36.20[e]	116.87[e]	197.85[e]	41.36[e]	138.16[e]	236.88[e]	54.29[e]	192.79[e]	338.66[e]
37.81[f]	122.17[f]	205.62[f]	43.25[f]	144.58[f]	246.40[f]	56.93[f]	202.15[f]	352.90[f]
36.25[g]	116.89[g]	198.10[g]	41.01[g]	137.03[g]	235.21[g]	53.47[g]	189.67[g]	332.87[g]
36.12[h]	116.81[h]	197.46[h]	40.85[h]	136.93[h]	234.44[h]	53.25[h]	189.52[h]	331.73[h]
37.73[i]	122.10[i]	205.21[i]	42.74[i]	143.30[i]	243.85[i]	55.86[i]	198.73[i]	345.56[i]
32.63[j]	102.43[j]	174.29[j]	37.36[j]	121.95[j]	209.96[j]	49.03[j]	171.32[j]	301.43[j]
32.52[k]	102.36[k]	173.72[k]	37.22[k]	121.86[k]	209.26[k]	48.84[k]	171.19[k]	300.39[k]
33.87[l]	107.01[l]	180.62[l]	38.85[l]	127.54[l]	217.73[l]	51.15[l]	179.51[l]	312.99[l]
32.49[m]	101.00[m]	171.69[m]	37.19[m]	120.21[m]	206.74[m]	------	------	------
30.24[n]	105.97[n]	181.85[n]	35.41[n]	128.37[n]	222.65[n]	48.25[n]	185.79[n]	328.93[n]
29.93[o]	105.78[o]	180.40[o]	35.09[o]	128.15[o]	220.99[o]	47.87[o]	185.52[o]	326.80[o]
33.67[p]	118.33[p]	198.48[p]	39.07[p]	142.23[p]	241.65[p]	52.38[p]	202.91[p]	353.14[p]
31.06[q]	101.59[q]	173.44[q]	39.35[q]	130.99[q]	224.90[q]	59.91[q]	208.59[q]	363.59[q]
30.76[r]	101.41[r]	172.10[r]	38.98[r]	130.76[r]	223.22[r]	59.42[r]	208.27[r]	361.13[r]
34.43[s]	113.02[s]	188.80[s]	43.41[s]	145.14[s]	244.09[s]	65.30[s]	228.67[s]	391.48[s]
32.49[t]	101.00[t]	171.70[t]	37.19[t]	120.20[t]	206.70[t]	------	------	------

a – Present work: Nusselt with friction factor of Filonenko , (1954) (Model of Prandtl and Taylor)
b – Present work: Nusselt with friction factor of Shah and Bhatti, (1987) (Model of Prandtl and Taylor)
c – Present work: Nusselt with friction factor of Dean, (1978) (Model of Prandtl and Taylor)
d – Present work: Nusselt with friction factor of Filonenko, (1954) (Model of Von Karman)
e – Present work: Nusselt with friction factor of Shah and Bhatti, (1987) (Model of Von Karman)
f – Present work: Nusselt with friction factor of Dean, (1978) (Model of Von Karman)
g – Present work: Nusselt with friction factor of Filonenko, (1954) (Model of Rannie)
h – Present work: Nusselt with friction factor of Shah and Bhatti, (1987) (Model of Rannie)
i – Present work: Nusselt with friction factor of Dean, (1978) (Model of Rannie)
j – Present work: Nusselt with friction factor of Filonenko, (1954) (Model of Reichardt and Spalding)
k – Present work: Nusselt with friction factor of Shah and Bhatti, (1987) (Model of Reichardt and Spalding)
l – Present work: Nusselt with friction factor of Dean, (1978) (Model of Reichardt and Spalding)
m – Özisik et al., (1989)
n – Gnielinsk's empirical correlation with Filonenko's, (1954) friction factor
o – Gnielinsk's empirical correlation with Shah and Bhatti's, (1987) friction factor
p – Gnielinsk's empirical correlation with Dean's, (1978) friction factor

q – Taylor's empirical correlation with Filonenko's, (1954) friction factor
r – Taylor's empirical correlation with Shah and Bhatti's, (1987) friction factor
s – Taylor's empirical correlation with Dean's, (1978) friction factor
t – Santos et al., (2001).

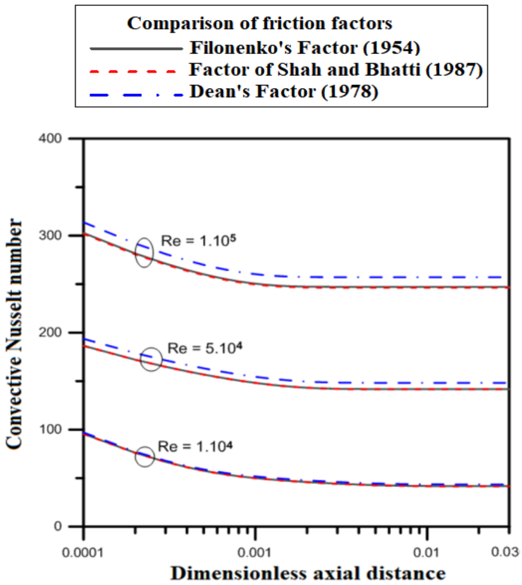

Figure 1. Local Nusselt number for turbulent flow between flat plates, considering Pr = 1, Prt = 1 and turbulence model of Prandtl and Taylor (Flat plates).

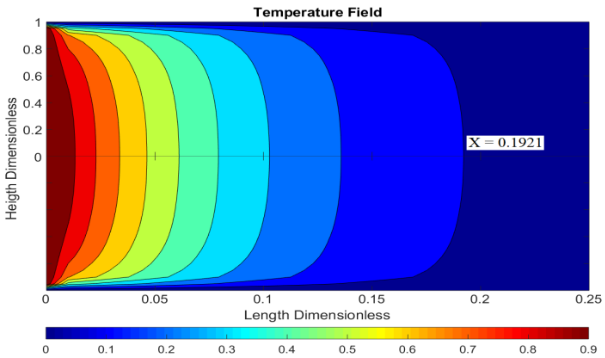

Figure 2. Temperature field for turbulent flow between flat plates, considering Pr = 1, Prt = 1, Re = 1.10^4 and friction factor of Filonenko (1954).

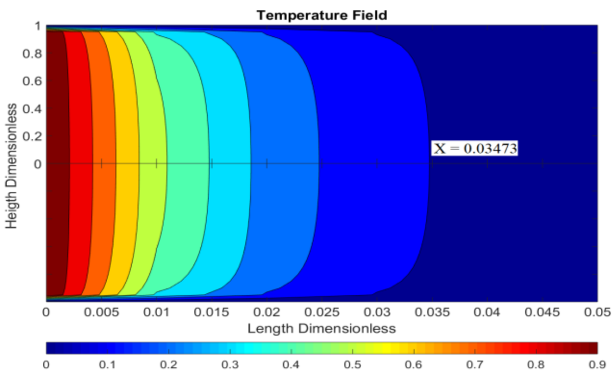

Figure 3. Temperature field for turbulent flow between flat plates, considering Pr = 1, Prt = 1, Re = 1.10^5 and friction factor of Filonenko, (1954).

The Table 2 shows the influence that the friction factor and the Reynolds and Prandtl numbers exert on the prediction of the dimensionless thermal input length. In the table 2 analyzes the case of flow between flat plates. In table it is considered the combination of different numbers of Reynolds (1.10^4, 5.10^4 and 1.10^5), different numbers of Prandtl (0.72, 1 and 2) and different correlations for the friction factor Dean, (1978), Filonenko, (1954) and Bhatti-Shah, (1987). The results presented allow to estimate, for each investigated situation, a trend value for the dimensionless length of thermal development, which can represent extremely important information in practical situations.

Table 2. Dimensionless length of thermal development for the turbulent flow between flat plates.

Flat Plates - $Pr_t = 1$								
$Pr = 0.72$			$Pr = 1$			$Pr = 2$		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
0.2100[a]	0.0676[a]	0.0399[a]	0.1921[a]	0.0588[c]	0.0347[a]	0.1670[a]	0.0471[a]	0.0266[a]
0.2108[b]	0.0676[b]	0.0400[b]	0.1929[b]	0.0588[b]	0.0349[b]	0.1676[b]	0.0471[b]	0.0267[b]
0.2022[c]	0.0649[c]	0.0388[c]	0.1847[c]	0.0565[c]	0.0333[c]	0.1598[c]	0.0451[c]	0.0256[c]
0.2189[d]	0.0712[d]	0.0428[d]	0.1910[d]	0.0599[d]	0.0360[d]	0.1447[d]	0.0432[d]	0.0257[d]
0.2197[e]	0.0712[e]	0.0429[e]	0.1917[e]	0.0599[e]	0.0361[e]	0.1453[e]	0.0433[e]	0.0257[e]
0.2109[f]	0.0683[f]	0.0411[f]	0.1838[f]	0.0577[f]	0.0348[f]	0.1388[f]	0.0410[f]	0.0248[f]
0.2204[g]	0.0713[g]	0.0429[g]	0.1942[g]	0.0606[g]	0.0364[g]	0.1480[g]	0.0441[g]	0.0262[g]
0.2212[h]	0.0713[h]	0.0430[h]	0.1950[h]	0.0606[h]	0.0365[h]	0.1486[h]	0.0441[h]	0.0262[h]
0.2123[i]	0.0684[i]	0.0412[i]	0.1868[i]	0.0582[i]	0.0352[i]	0.1420[i]	0.0419[i]	0.0253[i]
0.2492[j]	0.0831[j]	0.0493[j]	0.2164[j]	0.0693[j]	0.0407[j]	0.1629[j]	0.0490[j]	0.0286[j]
0.2500[k]	0.0832[k]	0.0494[k]	0.2171[k]	0.0694[k]	0.0409[k]	0.1635[k]	0.0490[k]	0.0287[k]
0.2407[l]	0.0794[l]	0.0479[l]	0.2086[l]	0.0666[l]	0.0394[l]	0.1566[l]	0.0471[l]	0.0278[l]

a –Dimensionless length of thermal development with friction factor of Filonenko, (1954) (Model of Prandtl and Taylor)
b – Dimensionless length of thermal development with friction factor of Shah and Bhatti, (1987) (Model of Prandtl and Taylor)
c – Dimensionless length of thermal development with friction factor of Dean, (1978) (Model of Prandtl and Taylor)
d –Dimensionless length of thermal development with friction factor of Filonenko, (1954) (Model of Von Karman
e – Dimensionless length of thermal development with friction factor of Shah and Bhatti, (1987) (Model of Von Karman)
f – Dimensionless length of thermal development with friction factor of Dean, (1978) (Model of Von Karman)
g –Dimensionless length of thermal development with friction factor of Filonenko, (1954) (Model of Rannie)
h – Dimensionless length of thermal development with friction factor of Shah and Bhatti, (1987) (Model of Rannie)
i – Dimensionless length of thermal development with friction factor of Dean, (1978) (Model of Rannie)
j –Dimensionless length of thermal development with friction factor of Filonenko, (1954) (Model of Reichardt and Spalding)
k – Dimensionless length of thermal development with friction factor of Shah and Bhatti, (1987) (Model of Reichardt and Spalding)
l – Dimensionless length of thermal development with friction factor of Dean, (1978) (Model of Reichardt and Spalding)

In the present study the thermal input length is defined as the maximum axial length required for the fluid to reach its final temperature with a margin of 10% relative difference. In practical situations this information may be relevant in the dimensioning process of the thermal equipment.

As predicted, the results differ from each other, but not with a very significant difference. Through this table it is possible to estimate, for a specified situation, the dimensional length of thermal development through equation 5a, that is, it is possible to find how many "meters" of duct are necessary to reach the thermal development.

5. FINAL CONSIDERATIONS

It is concluded from the analysis of the results obtained that the simultaneous application of the CITT and GITT is effective in solving the problem proposed, since the presented formulation was validated with the results found in the specialized literature. In this way, the objectives were reached satisfactorily, where the influence of the Reynolds and Prandtl numbers on the development of the thermal field and the local Nusselt number was shown. The analysis made in the present work is of extreme relevance, since the study of turbulence assumes great importance in engineering swing tothe great number of practical applications in which it is present. In applied areas such as heat exchanger design, reactor engineering and power engineering, laminar flow is an exception instead of the rule.

6. REFERENCES

BHATTI, M.S. AND SHAH, R.K., 1987. “Turbulent and Transition Flow Convective Heat Transfer in Ducts”, in: Kakaç, S., Shah, R.K. and Aung, W. (Eds.), Handbook of Single-Phase Convective Heat Transfer, John Wiley, New York.
BROWN, D.M., SANTOS, C.A.C., COTTA, R.M. AND KAKAÇ, S., 1997. “Analysis of Steady Forced Convection in Thermally Developing Turbulent Duct Flows, International Journal of Numerical Methods for Heat and Fluid Flow, 7, 424-437.
COTTA, R.M., 1998."The Integral Transform Method in Thermal and Fluid”. Science and Engineering, Begell House Inc, NY, USA.
COTTA, R.M.,1993. “Integral Transform in Computational Heat and Fluid Flow”. CRC Press, Boca Raton.
DEAN, R.B., 1978. “Reynolds Number Dependence of Skin Friction and other Bulk Flow Variables in Two-Dimensional Rectangular Duct Flow”, Journal of Fluids Engineering, 100, 215-223.
FILONENKO, G.K., 1954. “Hydraulic Resistance in Pipes (in Russian)”, Teploenergetika, 1, 40-44.
GNIELINSKI, V., 1976. “New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow”, International Chemical Engineering, 16, 359-368.
KAKAÇ, S. YENER, Y. W. and Pramuanjaroenkij, A., 2014. “Convective Heat Transfer”, CRC Press, 3rd edition, New York.
KAYS, W.M. AND CRAWFORD, M.E., 1980. “Convective Heat and Mass Transfer”, McGraw-Hill, New York.
KESTIN, J. AND RICHARDSON, P. D., 1963. “Heat Transfer Across Turbulent, Incompressible Boundary Layers”, Int. J. Heat Mass Transfer, Vol. 6, pp. 147-189.
MIKHAILOV, M. D. and ÖZISIK, M. N., 1984. “Unified Analysis and Solutions of Heat and Mass Diffusion”, John Wiley, New York.
MIKHAILOV, M. D. and VULCHANOV, N. L., 1983. “Computational procedure for Sturn-Liouville problems”, Journal of computational Physics, v. 5, 323-336.
NIKURADSE, J. (1950). Laws of flow in rough pipes, NACA Tech. Memo. 1292, 62 pp. (translated from German).
NOTTER, R.H. and SLEICHER, C.A. (1971) The Eddy Diffusivity in the Turbulent Boundary Layer near a Wall, Chemical Engineering Science, 26, 161-171.
NOTTER, R.H. and SLEICHER, C.A. (1972) A Solution to the Turbulent Graetz Problem - III. Fully Developed and Entry Region Heat Transfer Rates, Chemical Engineering Science, 27, 2073-2093.
ÖZISIK, M.N., COTTA, R.M. and KIM, W.S. (1989). " Heat Transfer in Turbulent Forced Convection between Parallel-Plates”, The Canadian Journal of Chemical Engineering, 67, 771-776.
PLETCHER, R. H., 1988. “Progress in Turbulent Forced Convection”, Journal of Heat Transfer, Vol. 110, pp. 1129-1144.
PRANDTL,L., 1910. “Eine Beziehung zwischen, Warmeaustausch und Stromungswiderstand der Flussigkeit”, Z. Physik, Vol.11, 1910, pp.1072-1078.
RANNIE, W.D., 1956. “Heat Transfer in Turbulent Shear Flow”, J. Aero. Sci., Vol. 23, pp. 485-489.
REICHARDT, H., 1951. “Volstandige Darstellung der Turbulenten Geschwindigkeitsverteilung in Glatten Leitungen”, Zeitschrift fur Angewandte Mathematik und Mechanik, 31, 208-219.
SANTOS, C.A.C., BROWN, D.M., KAKAÇ, S. AND COTTA, R.M., 1995. “Analysis of Unsteady Forced Convection in Turbulent Duct Flow”, Journal of Thermophysics and Heat Transfer, Vol. 9, pp. 508-515.
SANTOS, C.A.C.; QUARESMA, J.N.N. and LIMA, J. A., 2001. “Convective Heat Transfer in Ducts: the Integral Transform Approach”, 348 p., E-Papers, ABCM Mechanical Sciences Series, Rio de Janeiro, Brazil.
SCHLICHTING,H., 1960. “Boundary Layer Theory”, translated by J. Kestin (4th ed.), McGraw-Hill, New York.
SHIBANI, A.S. and ÖZISIK, M.N. (1977) A Solution to Heat Transfer in Turbulent Flow between Parallel Plates, International Journal of Heat and Mass Transfer, 20, 565-573.
SLEICHER, C.A., NOTTER, R.H. and CRIPPEN, M.D. (1970) A Solution of Turbulent Graetz Problem by Matched Asymptotic Expansions - I. The Case of Uniform Wall Temperature, Chemical Engineering Science, 25, 845-857.
SPALDING, D.B., 1961. “Heat Transfer to a Turbulent Stream from a Surface with a Step-Wise Discontinuity in Wall Temperature”, Proceedings of the International Conference of Developments in Heat Transfer, ASME, Part II, pp. 439-446, Boulder, USA.
TAYLOR, G.I., 1916. “Conditions at the surface of a hot body exposed to the Wind”, British Advisory Committe for Aeronautics, 2-R & M, N° 272, pp. 423-429.
VON KARMAN, TH., 1939. “The Analogy between Fluid Friction and Heat Transfer”, Trans. Amer. Soc. Mech. Engrs., Vol.61, pp. 705-710.

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