Escola de Primavera de Transição e Turbulência
Blumenau 2020

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

12th Spring School on Transition and Turbulence
September 21st-25th, 2020, Blumenau, SC, Brazil

EPTT-2020-0129
TURBULENT KINETIC ENERGY ANALYSIS IN TWO-DIMENSIONAL LID-DRIVEN CAVITY FLOW AT Re=100,000

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Abstract. *Simulation results based on the lattice Boltzmann method are shown for the turbulent lid-driven two-dimensional cavity flow at Reynolds number 100,000. We make use of a boundary condition scheme that significantly improves stability for simulation of turbulent flows within the lattice Boltzmann method framework. Explicit expressions for the turbulent kinetic energy budget are presented and its evolution is studied in 2D turbulence. We also look at the evolution of enstrophy and palinstrophy.*
Keywords: *budget of turbulent kinetic energy, lattice Boltzmann method, lid-driven cavity*

1. INTRODUCTION

Turbulence is ubiquitous in nature and in industrial processes, and for many years has garnered interest as a fundamental problem for physics as well as mathematics and engineering (Chen *et al.*, 2003; Landau and Lifshitz, 1987). In many situations of practical interest, there is statistical non-homogeneity and anisotropy in the flow field, where highly nontrivial interactions between bulk flow and boundary layers emerge (Hegele Jr. *et al.*, 2018). Computational fluid dynamics (CFD) in its current developmental stage offers a wide array of elaborate mathematical tools and numerical methods capable of solving the Reyolds-averaged or filtered Navier-Stokes equations in domains discretized with meshes of high geometrical complexity, over a large span of flow regimes, from low Mach subsonic to hypersonic. However, fundamental studies of turbulence rely on the full resolution of all the spatial and temporal scales present in the turbulent flow field. This is achieved through direct numerical simulation (DNS), which is currently limited for all practical purposes to simple geometries. The lid-driven cavity flow is a simple geometry that lends itself to the study of multiple boundary layer interaction with the turbulent bulk flow. The flow is created by a solid wall moving tangentially to itself, sealing off the cavity. In this work we restrict our attention to two-dimensional turbulence, which features fundamental differences with respect to three-dimensional turbulence, owing to the lack of the vortex-stretching term in the vorticity equation. Hegele Jr. *et al.* (2018), on which this work is based on, introduces a novel implementation of boundary conditions in the regularized lattice Boltzmann method (LBM) and also shows that, at increasing Re, complex flow dynamics emerge in the lid-driven cavity.

One of the fundamental quantities of interest is the turbulent kinetic energy (TKE) and its budget. To the best of our knowledge, neither the implementation nor the equations of TKE balance, as shown here, are found in other work. It is our aim to fill in this research gap regarding the 2D lid-driven cavity flow and TKE budget.

The choice of simulation method is LBM, a CFD solution that is based on a discretazion of the Boltzmann equation over a lattice and that can be shown to be equivalent to the macroscopic conservation equations, in addition to having simple and efficient implementations (Krüger *et al.*, 2017).

The paper is divided as follows: §2 presents the lattice Boltzmann method, the boundary conditions and the turbulent kinetic energy balance equation. The results are presented and discussed in §3. In §4 we offer the conclusions and lay out future work.

2. METHODOLOGY

2.1 The lattice Boltzmann Method

The lattice Boltzmann method solves the particle distribution function. The density of particles are represented by the particle populations, also known as the discrete-particle distribution function $f_i(\mathbf{r}, t)$ according to Eq. (1), which is related to velocity $\mathbf{c}_i$ at position $\mathbf{r}$ and time t, in dimensionless units. Moreover, there are velocity sets that are used to solve Navier-Stokes (N-S) equations, denoted by DdQq (where d is the number of spatial dimensions and q is the velocity set number). In this work we use the D3Q19 implementation (Krüger *et al.*, 2017),

$$f_i(\mathbf{r}+\mathbf{c}_i, t+1) = f_i^{(eq)}(\mathbf{r}, t) + (1-\tau^{-1})\widehat{f}_i^{(neq)}(\mathbf{r}, t), \tag{1}$$

where $f_i^{(eq)}$ is the equilibrium particle distribution and $\widehat{f}_i^{(neq)}$ is the regularized non-equilibrium particle distribution and τ is the relaxation time. The equilibrium depends on the local quantities density, ρ, and fluid velocity.

In order to regularize the particle distribution function, Eq. (2) gives the conserved macroscopic variables as weighted moments of the particle distribution function, of zeroth-order, ρ, first-order, ρu_α and second order $\rho m_{\alpha\beta}^{(2)}$. $\delta_{\alpha\beta}$ term is the Kronecker delta, a_s is the scaling factor equal to $\sqrt{3}$ and g_i is the particle distribution function f_i or regularized particle distribution function $\hat{f}_i$ (Latt and Chopard, 2006; Montessori *et al.*, 2014),

$$\left\{\rho, \rho u_\alpha, \rho m_{\alpha\beta}^{(2)}\right\} = \sum_i g_i \left\{1, c_{i\alpha}, c_{i\alpha i\beta} - \delta_{\alpha\beta}/a_s^2\right\}. \tag{2}$$

The equilibrium particle distribution function $f^{(eq)}$, is expressed considering the second-order velocity expansion in the Hermite polynomials (Philippi *et al.*, 2006; Shan *et al.*, 2006),

$$f_i^{(eq)}(\mathbf{r}, t) = \rho w_i \left(1 + a_s^2 u_\alpha c_{i\alpha} + \frac{1}{2} a_s^4 u_\alpha u_\beta \mathscr{H}_{\alpha\beta,i}^{(2)}\right), \tag{3}$$

where w_i are the quadrature weights, which depend on the absolute value of the direction $\mathbf{c}_i$. The second-order moments are projected onto the velocity space, as we can see in the Eq. (4):

$$\widehat{f}_i^{(neq)}(\mathbf{r}, t) = \frac{1}{2}\rho w_i a_s^4 \left[m_{\alpha\beta}^{(2)} - u_\alpha u_\beta\right] \mathscr{H}_{\alpha\beta,i}^{(2)}. \tag{4}$$

Finally, the regularization procedure is completed as Eq. (5) (Mattila *et al.*, 2017; Coreixas *et al.*, 2017),

$$\widehat{f}_i(\mathbf{r}, t) = f_i^{(eq)}(\mathbf{r}, t) + \widehat{f}_i^{(neq)}(\mathbf{r}, t). \tag{5}$$

By itself, the regularization procedure of LBM, in spite of presenting larger stability, does not necessarily address boundary conditions (Hegele Jr. *et al.*, 2018). Thus, from regularization procedure of particles distribution function, boundary conditions may be found turning possible the definition and characterization of the desired flow modeling (Latt *et al.*, 2008; Malaspinas *et al.*, 2011). With the sum of still-unknown regularized particles at the boundaries node, it is achievable to obtain the second-order particle momentum from Eq. (6), which is also expressed with regularized particles distribution function, as shows Eq. (7).

$$\sum_{i\in I_s} f_i \mathscr{H}_{\alpha\beta,i}^{(2)} + \sum_{i\notin I_s} \widehat{f}_i \mathscr{H}_{\alpha\beta,i}^{(2)} = \rho m_{\alpha\beta}^{(2)}, \tag{6}$$

which is also expressed with regularized particles distribution function,

$$\sum_{i\in I_s} \widehat{f}_i \mathscr{H}_{\alpha\beta,i}^{(2)} + \sum_{i\notin I_s} \widehat{f}_i \mathscr{H}_{\alpha\beta,i}^{(2)} = \rho m_{\alpha\beta}^{(2)}. \tag{7}$$

In short, through second-order particle momentum decomposition to regularized particles distribution function (considering belonging and not belonging particles distribution to an incoming velocity set I_s) combined to Eq. (6), equivalence is established between the sum of f_i function (regularized and not-regularized), according to Eq. (8), leading to a set of six equations when using D3Q19 distribution.

$$\sum_{i\in I_s} f_i \mathscr{H}_{\alpha\beta,i}^{(2)} = \sum_{i\in I_s} \widehat{f}_i \mathscr{H}_{\alpha\beta,i}^{(2)}. \tag{8}$$

In a nutshell, LBM consists on collision and streaming. The collision is simply an algebraic local operator, that calculates zero-order momentum and the macroscopic velocity $\mathbf{u}$ to find equilibrium distributions $f_i^{(eq)}$ and post-collision. After that, the resulting distribution of post-collision is streamed to neighbouring nodes (Krüger *et al.*, 2017).

Hegele Jr. *et al.* (2018) guide that when considering Dirichlet boundaries, the velocity $\mathbf{u}$ is known, in principle. Then, having mass conservation during the process of particles collision, the boundaries to the 3D fluid flow, to each node, can be found by Eq. (9):

$$\sum_{i\in I_s} f_i(\mathbf{r},t) = \sum_{i\in O_s} f_i(\mathbf{r}+\mathbf{c}_i, t+1) = (1-\tau^{-1})\sum_{i\in O_s} \widehat{f}_i(\mathbf{r},t) + \tau^{-1}\sum_{i\in O_s} f_i^{(eq)}(\mathbf{r},t). \tag{9}$$

2.2 Turbulent Kinetic Energy Budget

The following treatment will consider the three-dimensional form of the evolution equations, for the sake of generality. When appropriate, equations will be further simplified to two-dimensional form. The evolution of TKE is given by (Pope, 2000),

$$\partial_t k + U_\alpha \partial_\alpha k + \partial_\alpha T_\alpha^{'} = \mathscr{P} - \epsilon, \tag{10}$$

where $k \equiv \frac{1}{2}\langle u_\alpha^{'} u_\alpha^{'}\rangle$ is the turbulent kinetic energy, $u_\alpha^{'} \equiv u_\alpha - U_\alpha$ is the fluctuating velocity and $U_\alpha \equiv \langle u_\alpha\rangle$. The $\langle\cdot\rangle$ operator is the ensemble average. The turbulent transport $T_\alpha^{'}$ is given by

$$T_\alpha^{'} = \frac{1}{2}\langle u_\alpha^{'} u_\beta^{'} u_\beta^{'}\rangle + \langle u_\alpha^{'} \frac{p^{'}}{\rho}\rangle - 2\nu\langle u_\beta^{'} s_{\alpha\beta}\rangle, \tag{11}$$

and the production of TKE $\mathscr{P}$ is given by

$$\mathscr{P} = \langle u_\alpha^{'} u_\beta^{'}\rangle \partial_\beta U_\alpha. \tag{12}$$

Finally, the dissipation rate of TKE, ϵ, is given by

$$\epsilon = 2\nu s_{\alpha\beta} s_{\alpha\beta}, \tag{13}$$

where $s_{\alpha\beta} = \frac{1}{2}\left(\partial_\alpha u_\beta^{'} + \partial_\beta u_\alpha^{'}\right)$ is the fluctuating rate-of-strain tensor and $p^{'} = p - \langle p\rangle$ is the fluctuating (modified) pressure.

In the equations above, the operator $\langle\cdot\rangle$ represents the ensemble average. However, in this work we are interested in the statistically stationary turbulence state, and under this condition we approximate the ensemble average of a property ϕ as a time-average over a interval of duration δ_t, during which the system is in a statistically stationary state (Hegele Jr. *et al.*, 2018).

$$\langle\phi\rangle(x,y,z) \equiv \frac{1}{\delta_t}\int_{t_i}^{t_i+\delta_t} \phi(x,y,z,t)dt. \tag{14}$$

In the following equations we write out all of the TKE terms explicitly. It will facilitate the understanding of this important energy budget.

2.2.1 Turbulent Kinetic Energy Term

The TKE can be written as,

$$k \equiv \frac{1}{2}\langle u_\alpha^{'} u_\alpha^{'}\rangle = \frac{1}{2}\langle (u_\alpha - U_\alpha)(u_\alpha - U_\alpha)\rangle, \tag{15}$$

then:

$$k = \frac{1}{2}\langle u_\alpha u_\alpha - 2u_\alpha U_\alpha + U_\alpha U_\alpha\rangle, \tag{16}$$

so:

$$k = \frac{1}{2}\left(\langle u_\alpha u_\alpha\rangle - 2\langle u_\alpha U_\alpha\rangle + \langle U_\alpha U_\alpha\rangle\right). \tag{17}$$

Assuming $\langle u_\alpha u_\alpha \rangle = \langle u^2 \rangle$, $\langle u_\alpha U_\alpha \rangle = \langle u_\alpha \rangle U_\alpha = U_\alpha U_\alpha$, and that $\langle U_\alpha U_\alpha \rangle = \langle U_\alpha \rangle \langle U_\alpha \rangle = U_\alpha U_\alpha = U^2$, because U does not depend on t. Thereby, Eq. (18) leads to:

$$k = \frac{1}{2}\left(\langle u^2 \rangle - U^2\right), \tag{18}$$

where explicitly goes for:

$$k = \frac{1}{2}\left(\langle u_x^2 \rangle + \langle u_y^2 \rangle + \langle u_z^2 \rangle - U_x^2 - U_y^2 - U_z^2\right). \tag{19}$$

As the term $\partial_t k = 0$, the other one related to k, $U_\alpha \partial_\alpha k$, is given by:

$$U_\alpha \partial_\alpha k = U_x \frac{\partial k}{\partial x} + U_y \frac{\partial k}{\partial y} + U_z \frac{\partial k}{\partial z}. \tag{20}$$

2.2.2 Turbulent Transport Term

To the $T_\alpha^{'}$ term, let us separate it in three terms, as:

$$A_\alpha = \frac{1}{2}\langle u_\alpha^{'} u_\beta^{'} u_\beta^{'} \rangle, \tag{21}$$

$$B_\alpha = \langle u_\alpha^{'} p^{'} \rangle \frac{1}{\rho}, \tag{22}$$

and

$$C_\alpha = -2\nu \langle u_\beta^{'} s_{\alpha\beta} \rangle, \tag{23}$$

where $T_\alpha^{'} = A + B + C$.

So, taking term A, it is possible to express mathematically as:

$$A_\alpha = \frac{1}{2}\langle (u_\alpha - U_\alpha)(u_\beta - U_\beta)(u_\beta - U_\beta) \rangle, \tag{24}$$

then:

$$A_\alpha = \frac{1}{2}\langle (u_\alpha - U_\alpha)(u_\beta u_\beta - 2u_\beta U_\beta + U_\beta U_\beta) \rangle, \tag{25}$$

so:

$$A_\alpha = \frac{1}{2}\langle u_\alpha u_\beta u_\beta + u_\alpha U_\beta U_\beta - 2u_\alpha u_\beta U_\beta - u_\beta u_\beta U_\alpha - U_\alpha U_\beta U_\beta + 2u_\beta U_\alpha U_\beta \rangle. \tag{26}$$

Using mathematical manipulation,

$$A_\alpha = \frac{1}{2}\left(\langle u_\alpha u_\beta u_\beta \rangle + \langle u_\alpha U_\beta U_\beta \rangle - 2\langle u_\alpha u_\beta U_\beta \rangle - \langle u_\beta u_\beta U_\alpha \rangle - \langle U_\alpha U_\beta U_\beta \rangle + 2\langle u_\beta U_\alpha U_\beta \rangle\right). \tag{27}$$

In an analogous way to what we assumed in before, we can simplify the parameters, so:

$$A_\alpha = \frac{1}{2}\left(\langle u_\alpha u^2 \rangle + U_\alpha U^2 - 2\langle u_\alpha u_\beta \rangle U_\beta - U_\alpha u^2 - U_\alpha U^2 + 2U_\alpha U^2\right), \tag{28}$$

cancelling some terms, it follows:

$$A_\alpha = \frac{1}{2}\left(\langle u_\alpha u^2 \rangle - 2\langle u_\alpha u_\beta \rangle U_\beta - U_\alpha u^2 + 2U_\alpha U^2\right). \tag{29}$$

Continuing, we now take term B, that can be expressed as:

$$B_\alpha = \langle (u_\alpha - U_\alpha)(p - \langle p \rangle) \rangle \frac{1}{\rho}, \tag{30}$$

as for this work ρ is always equal to 1 (Ma number is too much lower than 1) and making use algebra:

$$B_\alpha = \langle u_\alpha p - U_\alpha p - u_\alpha \langle p \rangle + U_\alpha \langle p \rangle \rangle, \tag{31}$$

then:

$$B_\alpha = \langle u_\alpha p\rangle - \langle U_\alpha p\rangle - \langle u_\alpha \langle p\rangle\rangle + \langle U_\alpha \langle p\rangle\rangle, \tag{32}$$

and after mathematical manipulations:

$$B_\alpha = \frac{\langle u_\alpha p\rangle}{3} - U_\alpha \langle p\rangle. \tag{33}$$

Finally, for term C we get:

$$C_\alpha = -2\nu\langle (u_\beta - U_\beta)\frac{1}{2}\left(\partial_\alpha u_\beta^{'} + \partial_\beta u_\alpha^{'}\right)\rangle, \tag{34}$$

so:

$$C_\alpha = -2\nu\langle (u_\beta - U_\beta)\frac{1}{2}\left(\partial_\alpha (u_\beta - U_\beta) + \partial_\beta (u_\alpha - U_\alpha)\right)\rangle, \tag{35}$$

then:

$$C_\alpha = -\nu\langle (u_\beta\partial_\alpha(u_\beta - U_\beta) - U_\beta\partial_\alpha(u_\beta - U_\beta) + u_\beta\partial_\beta(u_\alpha - U_\alpha) - U_\beta\partial_\beta(u_\alpha - U_\alpha)\rangle, \tag{36}$$

making the distributive:

$$C_\alpha = -\nu\langle u_\beta\partial_\alpha u_\beta - u_\beta\partial_\alpha U_\beta - U_\beta\partial_\alpha u_\beta + U_\beta\partial_\alpha U_\beta + u_\beta\partial_\beta u_\alpha - u_\beta\partial_\beta U_\alpha - U_\beta\partial_\beta u_\alpha + U_\beta\partial_\beta U_\alpha\rangle, \tag{37}$$

applying mean parameter in each term:

$$\begin{aligned} C_\alpha = &- \nu(\langle u_\beta\partial_\alpha u_\beta\rangle - \langle u_\beta\partial_\alpha U_\beta\rangle - \langle U_\beta\partial_\alpha u_\beta\rangle + \langle U_\beta\partial_\alpha U_\beta\rangle + \langle u_\beta\partial_\beta u_\alpha\rangle - \langle u_\beta\partial_\beta U_\alpha\rangle - \langle U_\beta\partial_\beta u_\alpha\rangle \\ &+ \langle U_\beta\partial_\beta U_\alpha\rangle). \end{aligned} \tag{38}$$

In order to facilitate, we take term by term in Eq. (38), where:

$$\langle u_\beta\partial_\alpha u_\beta\rangle = \frac{1}{2}\partial_\alpha\langle u_\beta u_\beta\rangle = \frac{1}{2}\partial_\alpha\langle u^2\rangle, \tag{39}$$

$$-\langle u_\beta\partial_\alpha U_\beta\rangle = -\langle u_\beta\rangle\partial_\alpha U_\beta = -U_\beta\partial_\alpha U_\beta = -\frac{1}{2}\partial_\alpha U^2, \tag{40}$$

$$-\langle U_\beta\partial_\alpha u_\beta\rangle = -U_\beta\partial_\alpha\langle u_\beta\rangle = -U_\beta\partial_\alpha U_\beta = -\frac{1}{2}\partial_\alpha U^2, \tag{41}$$

$$\langle U_\beta\partial_\alpha U_\beta\rangle = U_\beta\partial_\alpha U_\beta = \frac{1}{2}\partial_\alpha U^2, \tag{42}$$

$$\langle u_\beta\partial_\beta u_\alpha\rangle = \partial_\beta\langle u_\beta u_\alpha\rangle, \tag{43}$$

$$-\langle u_\beta\partial_\beta U_\alpha\rangle = -\langle u_\beta\rangle\partial_\beta U_\alpha = -U_\beta\partial_\beta U_\alpha, \tag{44}$$

$$-\langle U_\beta\partial_\beta u_\alpha\rangle = -U_\beta\partial_\beta\langle u_\alpha\rangle = -U_\beta\partial_\beta U_\alpha, \tag{45}$$

$$\langle U_\beta\partial_\beta U_\alpha\rangle = U_\beta\partial_\beta U_\alpha. \tag{46}$$

It is possible to observe that Eq. (40) with Eq. (41), and Eq. (44) with Eq. (46) are cancelling themselves. So, Eq. (38) is shown as:

$$C_\alpha = -\nu\left(\frac{1}{2}\partial_\alpha\langle u^2\rangle - \frac{1}{2}\partial_\alpha U^2 + \partial_\beta\langle u_\beta u_\alpha\rangle - U_\beta\partial_\beta U_\alpha\right). \tag{47}$$

2.3 Production of Turbulence Term

Following the TKE equation, in this subsection we are going to explicit the $\mathscr{P}$ term, that can be expressed by:

$$\mathscr{P} = \langle (u_\alpha - U_\alpha)(u_\beta - U_\beta) \rangle \partial_\beta U_\alpha, \tag{48}$$

so:

$$\mathscr{P} = \langle (u_\alpha u_\beta - U_\alpha u_\beta - u_\alpha U_\beta + U_\alpha U_\beta) \rangle \partial_\beta U_\alpha, \tag{49}$$

applying mean parameter to each term:

$$\mathscr{P} = (\langle u_\alpha u_\beta \rangle - \langle U_\alpha u_\beta \rangle - \langle u_\alpha U_\beta \rangle + \langle U_\alpha U_\beta \rangle) \partial_\beta U_\alpha, \tag{50}$$

then:

$$\mathscr{P} = (\langle u_\alpha u_\beta \rangle - U_\alpha \langle u_\beta \rangle - \langle u_\alpha \rangle U_\beta + U_\alpha U_\beta) \partial_\beta U_\alpha, \tag{51}$$

and simplifying:

$$\mathscr{P} = (\langle u_\alpha u_\beta \rangle - U_\alpha U_\beta) \partial_\beta U_\alpha. \tag{52}$$

2.3.1 Dissipation of Turbulent Kinetic Energy Term

To express the dissipation term, from Eq. (13), we take:

$$\epsilon = 2\nu \frac{1}{2} \left[\partial_\alpha (u_\beta - U_\beta) + \partial_\beta (u_\alpha - U_\alpha) \right] \frac{1}{2} \left[\partial_\alpha (u_\beta - U_\beta) + \partial_\beta (u_\alpha - U_\alpha) \right], \tag{53}$$

so:

$$\epsilon = \nu \frac{1}{2} (\partial_\alpha u_\beta - \partial_\alpha U_\beta + \partial_\beta u_\alpha - \partial_\beta U_\alpha)(\partial_\alpha u_\beta - \partial_\alpha U_\beta + \partial_\beta u_\alpha - \partial_\beta U_\alpha), \tag{54}$$

then:

$$\begin{aligned} \epsilon =& \nu \frac{1}{2} (\partial_\alpha u_\beta \partial_\alpha u_\beta - \partial_\alpha u_\beta \partial_\alpha U_\beta + \partial_\alpha u_\beta \partial_\beta u_\alpha - \partial_\alpha u_\beta \partial_\beta U_\alpha - \partial_\alpha U_\beta \partial_\alpha u_\beta + \partial_\alpha U_\beta \partial_\alpha U_\beta - \partial_\alpha U_\beta \partial_\beta u_\alpha \\ &+ \partial_\alpha U_\beta \partial_\beta U_\alpha + \partial_\beta u_\alpha \partial_\alpha u_\beta - \partial_\beta u_\alpha \partial_\alpha U_\beta + \partial_\beta u_\alpha \partial_\beta u_\alpha - \partial_\beta u_\alpha \partial_\beta U_\alpha - \partial_\beta U_\alpha \partial_\alpha u_\beta + \partial_\beta U_\alpha \partial_\alpha U_\beta \\ &- \partial_\beta U_\alpha \partial_\beta u_\alpha + \partial_\beta U_\alpha \partial_\beta U_\alpha), \end{aligned} \tag{55}$$

where simplifying we have:

$$\begin{aligned} \epsilon =& \nu \frac{1}{2} (\partial_\alpha u_\beta \partial_\alpha u_\beta - 2\partial_\alpha u_\beta \partial_\alpha U_\beta + 2\partial_\alpha u_\beta \partial_\beta u_\alpha - 2\partial_\alpha u_\beta \partial_\beta U_\alpha + \partial_\alpha U_\beta \partial_\alpha U_\beta - 2\partial_\alpha U_\beta \partial_\beta u_\alpha + 2\partial_\alpha U_\beta \partial_\beta U_\alpha \\ &+ \partial_\beta u_\alpha \partial_\beta u_\alpha - 2\partial_\beta u_\alpha \partial_\beta U_\alpha + \partial_\beta U_\alpha \partial_\beta U_\alpha). \end{aligned} \tag{56}$$

3. PRELIMINARY RESULTS

Following Hegele Jr. *et al.* (2018), we set a constant tangential velocity equal to $\mathbf{u} = (u_L, 0, 0)$ applied at the top of the cubic cavity and on the other five faces of the cavity we set the velocity to zero. The spatial discretization is given by $N_x = N_z = 2049$ and $N_y = 1$. This allows for the implementation of a two-dimensional flow within the more general framework of a three-dimensional code. For the lid velocity, we consider $u_L = 0.1c_s$, where $c_s = 1/\sqrt{3}$ is the sound speed in the fluid, avoiding compressibility effects by keeping the Mach number low. The relaxation time is tuned to set the Reynolds number through the viscosity $\nu = (\tau - 1/2)/a_s^2$. The pressure p is given by $p = c_s^2 \rho$. The flow Reynolds number is 100,000.

When the Reynolds number is low (laminar) the flow in the cavity remains steady. But as Re increases considerably, going through transition, the normal velocity traces (u along z and w along x) show that the center of the vortex created by the fluid moves towards the cavity center. which can be seen in Fig. 1. We note that there is already, even with 5×10^6 time steps, considerable turbulence in the cavity, owing to the high-Reynolds number.

Kinetic energy is steadily increasing as is shown in Fig. 2. No sign of a statistically stationary state is seen. The evolution of TKE will depend on the relative magnitude of the terms in Eq. (10). Since there is no term associated with vortex-stretching in 2D turbulence, vorticity can only be generated along the solid boundaries, diffusing towards the bulk flow. A snapshot of the vorticity field is shown in Figure 3 after 5×10^6 time steps. The irregular pattern of many vortices in the cavity indicate the complex nature of turbulent flow. Analysis of this flow will be completed as the simulation progresses.

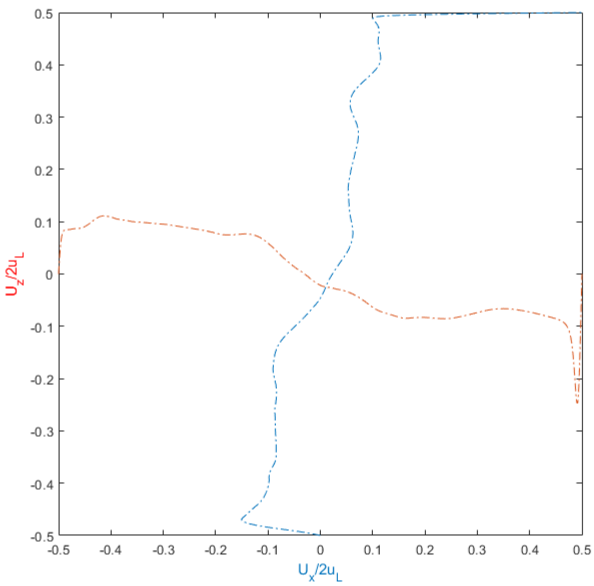

Figure 1. Normal velocity traces along the centerlines of the x and z

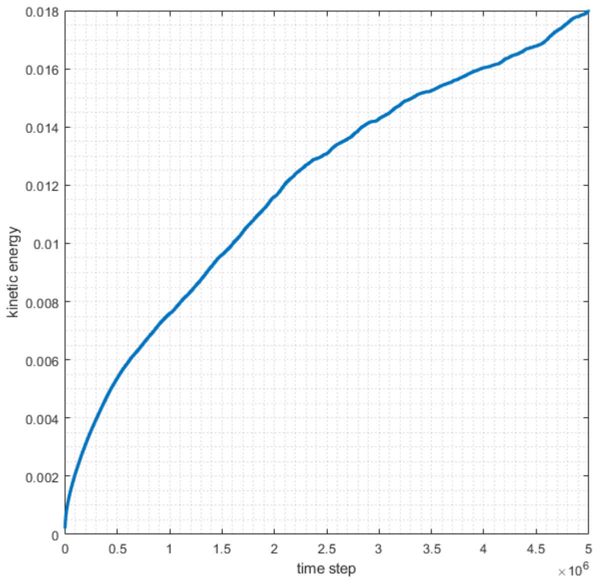

Figure 2. Kinetic energy at 5×10^6 time steps

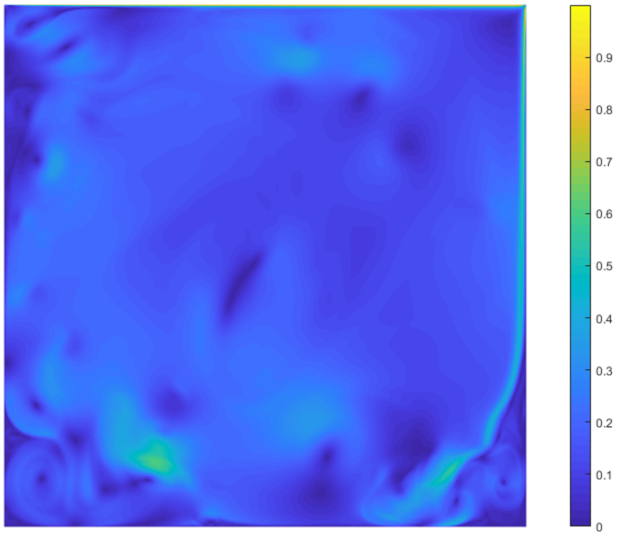

Figure 3. Vorticity field of the infinite span lid-driven cavity at 5×10^6 time steps

4. CONCLUSIONS

We note that the LBM boundary condition implementation adopted in this simulation with $Re = 100,000$ is effectively stable, even considering a large mesh resolution. In spite of the simulation having not reached a statistically stationary state, we estimate it will be achieved with an additional 10×10^6 simulation time steps.

After the statistically stationary state is reached, we can evaluate the TKE, in order to check the consistency of the results, and also analyse enstrophy and palinstrophy.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge PROGRAMA UNIEDU/FUMDES PÓS-GRADUAÇÃO, for the financial support for this study.

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