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INFLUENCE OF SPAN-WISE PERIODICITY ON THE LINEAR STABILITY OF ROSSITER MODES IN AN OPEN CAVITY

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Abstract. *We have used the Linear Stability Theory (LST) to compute the bi-global modes of a subsonic compressible flow over an open rectangular cavity to map how the instability of these modes vary with the span-wise period length of the domain. An infinite-length domain represents the 2D case, which is ideal for the Rossiter modes. As the domain becomes shorter, 3D effects become stronger, which causes these modes to become less unstable. We attribute this decreased instability to destructive interference in the acoustic feedback mechanism as the domain becomes narrower.*

Keywords: *Biglobal stability analysis, Rossiter modes, Open cavity flow, Compressible subsonic flow*

1. INTRODUCTION

An open cavity is a canonical geometry that can be used to models the flow on different parts of an aircraft or other vehicles, from the gaps between a wing's main element and its flaps and slats to the open sunroof of a car.

The flow in such open cavity may present different oscillation patterns depending on its parameters. Rossiter modes, which are usually most unstable at medium to high-subsonic Mach numbers, are usually uniform in the span-wise direction, therefore, two-dimensional modes. Nevertheless, in a real-world situation, perfect two-dimensionality is nothing but an approximation, as real scenarios will have some kind of non-uniformity in the span-wise direction, which can be caused by either some three-dimensional phenomenon happening in tandem or by the simple fact that no geometry is infinite in this direction.

Figure 1 illustrates the cross-section of this geometry. Rossiter (1964) has described a type of oscillation mode that consists in four stages: (1) A disturbance present in the mixing layer travels downstream and is amplified by its spatial instabilities. (2) This disturbance hits the cavity's trailing edge and creates acoustic emissions. (3) These acoustic waves travel back through the cavity to the leading edge. (4) These waves trigger new disturbances in the mixing layer when they hit the leading edge. These modes depend on compressible effects to be accurately predicted, due to their acoustic feedback mechanism. The number of the mode indicates how many vortices are present in the mixing layer at once. Despite being a non-linear mode, it is possible to find a linear equivalent to it with a global instability analysis (Mathias and Medeiros, 2019).

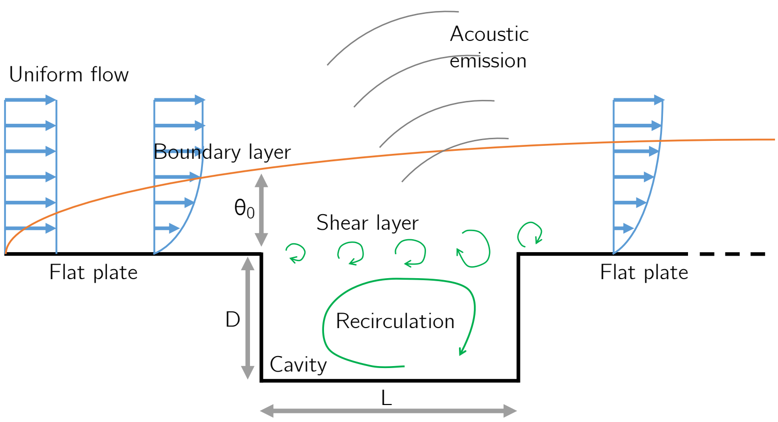

Figure 1. Cross-section of an open-cavity.

In this work, we have used the Linear Stability Theory (LST) to compute the bi-global modes of a subsonic compressible flow over the open rectangular cavity to map how the instability of these modes vary with the span-wise period length of the domain. An infinite-length domain represents the 2D case, which is ideal for the Rossiter modes. As the domain becomes shorter, 3D effects become stronger, which causes these modes to become less unstable.

2. METHODS

The Linear Stability Theory (LST) analysis in this work is done by a time-stepping algorithm, as described by Eriksson and Rizzi (1985); Chiba (1998); Gómez *et al.* (2014). In summary, the algorithm adds small disturbances to the base flow and uses a flow simulation routine to simulate them for a short length of time. After enough orthogonal disturbances are used, the method builds a reduced order model and compute its eigenmodes, which can be easily related to the most unstable modes of the original system.

For the flow simulation, we use our Direct Numerical Simulation code (DNS), which is governed by the compressible Navier-Stokes equations. Some of its characteristics are listed below:

- Structured mesh;
- Mesh stretching to concentrate nodes in the boundary and mixing layers;
- 4th order Runge-Kutta for the time-stepping;
- 4th order spectral-like spatial differentiation (Lele, 1992);
- 10th order spatial anti-aliasing filter (Gaitonde and Visbal, 1998);
- Temporal low-pass filter (SFD) to allow reaching a base-flow even ate unstable conditions (Åkervik *et al.*, 2006);
- Buffer-zones at the open boundaries;
- Domain decomposition for parallel execution (Li and Laizet, 2010).

Details on our implementation of both routines and their validation can be found in Mathias and Medeiros (2018).

3. RESULTS

Our reference case is at $Ma = 0.5$, $Re_D = 1000$ and $L/D = 2$, the momentum thickness of the boundary layer at the leading edge is 1% of the cavity depth D. β is the span-wise wave-number which means that the period length in this direction is $2\pi/\beta$. The base flow is shown in Fig. 2. The maximum backwards flow velocity is 24% of the free flow.

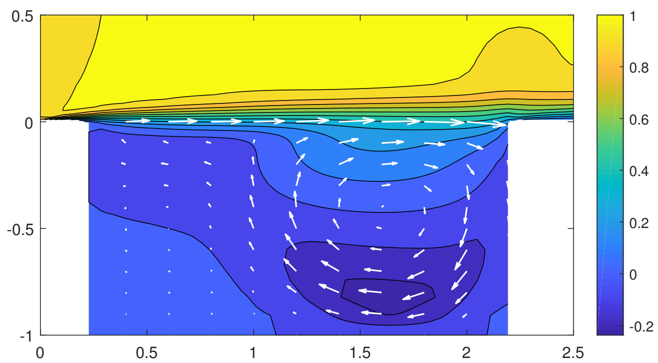

Figure 2. Contours of stream-wise velocity of the base flow.

This base flow is given as input to our LST routine, which computes the most unstable modes present in it. We sweep β values from 0 to 4π, where $\beta = 0$ represents the perfectly two-dimensional case and larger values indicate an increasingly three-dimensional flow.

Each mode in the flow is identified by an eigenvalue and an eigenfunction. The real part of the eigenvalue denotes the mode's stability: positive real parts indicate unstable modes. The imaginary part relates to its frequency. The eigenfunction indicates how and where the fluctuation happens in the flow. We can identify Rossiter modes by looking at their eigenfunction, which will contain both the disturbance in the mixing layer and the acoustic feedback wave inside the cavity. Figure 3 shows isocontours of the real and imaginary parts of the density fluctuation of Rossiter mode 2. In this flow, it is the most unstable mode, with an eigenvalue of $\sigma = 0.3233 \pm 2.7748i$. Its frequency is given by $f = \sigma_i/2\pi$, therefore, $f = 0.44$. The frequency for this mode as estimated by the empirical equation is $f = 0.42$ (Tam, 1976). This difference can be attributed to the different assumptions used by both approaches. Our LST analysis does not account for non-linear effects, such as the changes in the mean-flow caused by the vortex street in the mixing layer. On the other hand, this empirical equation does not account for some flow parameters, such as the incoming boundary layer thickness.

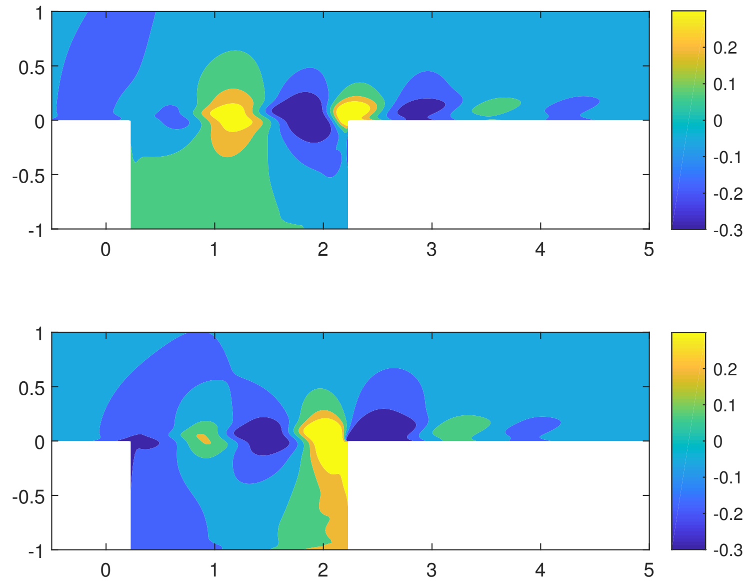

Figure 3. Isocountours of density of Rossiter mode 2. (a) Real part. (b) Imaginary part.

Figure 4 shows the eigenvalues of the flow for various values of β. The left-had side of the image show them in the complex plane. The right-hand side shows their real and imaginary part as a function of β. As β increases, the modes become less unstable. The highest value of β to present unstable Rossiter modes was $\beta = 0.5\pi$, in which the domain width is twice the cavity depth and equal to its length. As the domain became narrower, the frequencies of those mode have also slightly increased.

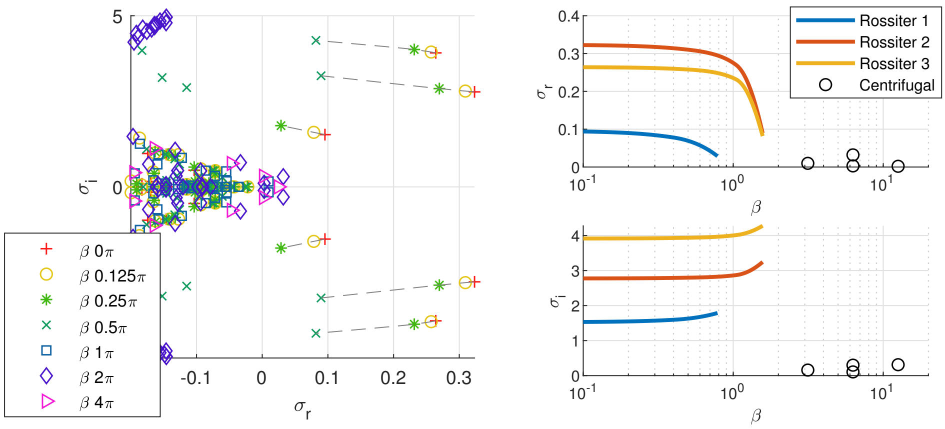

Figure 4. Eigenvalues from the span-wise wave number sweep. (a) Spectra in the complex plane. (b) Real and imaginary parts of the eigenvalues.

We attribute this reduction in instability, at least partially, to the acoustic feedback mechanism of these modes. In a 2D case, the whole span of the cavity trailing edge is acting as a linear sound source for the acoustic feedback. In a span-wise modulated case, the sound source is no longer linear and infinite, it is instead a series of alternated sources which, in the far-field, would cancel out each other. As β increases, these sources move closer to each other, increasing the acoustic decay from the trailing edge to the leading edge, thus reducing the disturbance amplitude at the beginning of the mixing layer. This effect can be observed in Fig.5, which brings the amplitude of the eigenfunction of pressure at the bottom of the cavity for Rossiter mode 2 at different values of β. The amplitude is normalized by its value at the cavity trailing edge. Notice that as the span-wise wavenumber is increased, the magnitude of the amplitude that reaches the leading edge of the cavity becomes smaller. The $\beta = \pi/2$ case, with had a sharp decline in instability as per Fig. 4 has also had a sharp decline in the acoustic wave amplitude as it reached the leading edge. There are also other factors at play here, as the eigenfunction cannot isolate one stage of the Rossiter loop from the others, but we believe that the interference of opposing sound sources plays an important role in reducing the Rossiter modes instability as the span-wise wave number increases.

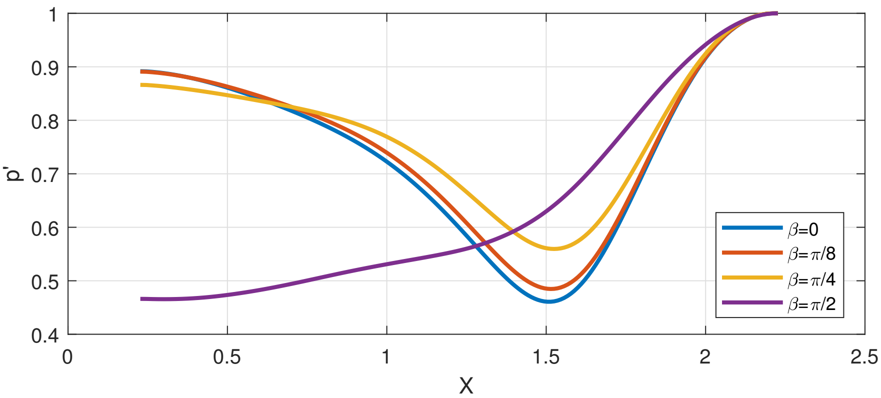

Figure 5. Amplitude of the eigenfunction of pressure at the bottom of the cavity for Rossiter mode 2 at different values of β.

At higher values of β, other types of modes begin to appear, those are centrifugal modes, which as caused by Görtler instabilities in the main recirculation area of the cavity (Brès and Colonius, 2008). Figure 6 compares the eigenfunctions of a Rossiter mode to that of a centrifugal mode.

This analysis is done in a linear framework, but may shed some light at more complex situations such as a three-dimensional flow with non-linear effects. In a real situation, the Rossiter structures might not be completely uniform in the span-wise direction as other three-dimensional effects may be at play, such as centrifugal modes.

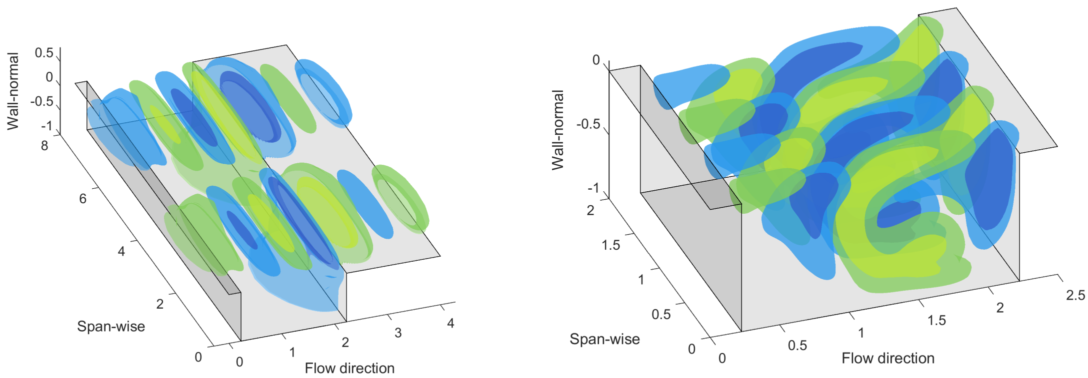

Figure 6. (a) Isosurfaces of density for Rossiter mode 2 at $\beta = \pi/4$. (b) Isosurfaces of span-wise velocity of a centrifugal mode at $\beta = 2\pi$.

4. CONCLUSION

We have used our LST routine to compute the instability levels of Rossiter modes considering various values of span-wise wave numbers. The two-dimensional case ($\beta = 0$) has resulted in the most unstable modes, which is to be expected, as these modes are essentially 2D. As the span-wise domain length was reduced, Rossiter modes were still unstable, but to a lesser degree. We can partially attribute this to the acoustic feedback mechanism. The linear stability results also indicate that there is a slight upwards shift in the mode's frequency as β increases.

5. ACKNOWLEDGMENTS

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REFERENCES

Åkervik, E., Brandt, L., Henningson, D.S., Hœpffner, J., Marxen, O. and Schlatter, P., 2006. “Steady solutions of the Navier-Stokes equations by selective frequency damping”. *Physics of Fluids*, Vol. 18, No. 6, pp. 68–102. ISSN 10706631. doi:10.1063/1.2211705. URL `http://scitation.aip.org/content/aip/journal/pof2/18/6/10.1063/1.2211705`.

Brès, G.A. and Colonius, T., 2008. “Three-dimensional instabilities in compressible flow over open cavities”. *Journal of Fluid Mechanics*, Vol. 599, pp. 309–339. ISSN 0022-1120. doi:10.1017/S0022112007009925. URL `http://www.journals.cambridge.org/abstract_S0022112007009925`.

Chiba, S., 1998. “Global Stability Analysis of Incompressible Viscous Flow”. *Journal of Japan Society of Computational Fluid Dynamics*, Vol. 7, No. 1, pp. 20–48.

Eriksson, L.E. and Rizzi, A., 1985. “Computer-aided analysis of the convergence to steady state of discrete approximations to the euler equations”. *Journal of Computational Physics*, Vol. 57, No. 1, pp. 90–128. ISSN 10902716. doi:10.1016/0021-9991(85)90054-3.

Gaitonde, D.V. and Visbal, M.R., 1998. “High-Order Schemes for Navier-Stokes Equations: Algorithm and Implementation Into FDL3DI”. Technical report, Wright-Patterson Air Force Base.

Gómez, F., Gómez, R. and Theofilis, V.V., 2014. “On three-dimensional global linear instability analysis of flows with standard aerodynamics codes”. *Aerospace Science and Technology*, Vol. 32, No. 1, pp. 223–234. ISSN 12709638. doi:10.1016/j.ast.2013.10.006. URL `http://linkinghub.elsevier.com/retrieve/pii/S1270963813001879`.

Lele, S.K., 1992. “Compact finite difference schemes with spectral-like resolution”. *Journal of Computational Physics*, Vol. 103, No. 1, pp. 16–42. ISSN 00219991. doi:10.1016/0021-9991(92)90324-R.

Li, N. and Laizet, S., 2010. “2DECOMP and FFT-A Highly Scalable 2D Decomposition Library and FFT Interface”. *Cray User Group 2010 conference*, pp. 1–13.

Mathias, M.S. and Medeiros, M., 2018. “Direct Numerical Simulation of a Compressible Flow and Matrix-Free Analysis of its Instabilities over an Open Cavity”. *Journal of Aerospace Technology and Management*, Vol. 10, pp. 1–13. ISSN 2175-9146. doi:10.5028/jatm.v10.949. URL `http://www.jatm.com.br/ojs/index.php/jatm/article/view/949`.

Mathias, M.S. and Medeiros, M.A.F.D., 2019. “Rossiter modes in a compressible open cavity: a comparison between the linear predictions and non-linear simulations”. *ERCOFTAC Bulletin 118*, pp. 16–19.

Rossiter, J.E., 1964. “Wind-tunnel experiments on the flow over rectangular cavities at subsonic and transonic speeds”. Technical Report Aeronautical Research Council Reports and Memoranda 3438, Ministry of Aviation, London. URL `http://repository.tudelft.nl/view/aereports/uuid:a38f3704-18d9-4ac8-a204-14ae03d84d8c/`.

Tam, C.K.W., 1976. “The acoustic modes of a two-dimensional rectangular cavity”. *Journal of Sound and Vibration*, Vol. 49, No. 3, pp. 353–364. ISSN 10958568. doi:10.1016/0022-460X(76)90426-0.

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