Vortex-Induced Vibration Model of Rigid Cylinders With 2 Degrees of Freedom Near A Fixed Wall Based On Wake Oscillator

Rafael Fehér, rafael.feher@ufabc.edu.br[1]
Juan Pablo Julca Avila, juan.avila@ufabc.edu.br[2]
Universidade Federal do ABC, Av. dos Estados, 5001 - Bangú, Santo André - SP, 09210-580

Resumo. *Quando uma estrutura em alto-mar é instalada próxima à uma fronteira, como o berço do mar, os vórtices desprendidos na região inferior dessa estrutura podem não ser completamente desenvolvidos, afetando o movimento dessa estrutura. No presente trabalho, um novo modelo de oscilador de fluido para prever as Vibrações Induzidas por Vórtices (VIV) de um cilindro rígido com dois graus de liberdade inicionalmente colocado próximo a uma parede fixa é proposto. Para modelar os efeitos da parede fixa no movimento do cilindro, dois coeficientes empíricos, β e η, são acoplados à equação do oscilador de fluido. Foram simuladas razões de espaçamentos, definidas como as razões entre a distância do cilindro à parede fixa, e, e o diâmetro do cilindro, D, de $0,75$ até 2 em um cilindro exposto a um fluxo de fluido. Os coeficientes empíricos foram calibrados utilizando-se dados experimentais. Após a calibração, utilizando-se regressão polinomial, as equações para se calcular cada um dos coeficientes empíricos em função da razão de espaçamento foram geradas. Os resultados de amplitude transversal reduzida e coeficiente de arrasto médio para cada razão de espaçamento são apresentados em função da velocidade reduzida e mostram-se estar em boa conformidade com os dados experimentais.*

Palavras chave: *Vibrações Induzidas Por Vórtices. Oscilador De Esteira. Interação Fluido-Estrutura. Vibrações Com Dois Graus de Liberdade.*

Abstract. *When an offshore structure is placed near a boundary, such as the seabed, the vortex shedding from the downstream side of this structure may be not fully developed, affecting the motion of the structure. In the present work, a new wake oscillator model to predict Vortex-Induced Vibrations (VIV) of a rigid cylinder with 2 Degrees of Freedom (2-DOF) initially placed near a fixed wall is proposed. To account for the effects of the fixed wall on the motion of the cylinder, two empirical coefficients, β and η, are coupled to the wake oscillator equation. Gap ratios, defined as the ratio between the distance of the fixed wall to the cylinder, e, and the diameter of the cylinder, D, varying from 0.75 to 2 were simulated on a cylinder exposed to a fluid flow. The empirical coefficients were calibrated using experimental data. After the calibration, using polynomial regression, the equations to calculate each empirical coefficient as function of the gap ratio were generated. The results of cross-flow reduced amplitude and mean drag coefficient for each gap ratio are presented in function of the reduced velocity and shows to be in good agreement with experimental data.*

Keywords: *Vortex-Induced Vibrations. Wake Oscillator. Fluid-Structure Interaction. Two Degrees of Freedom Vibrations*

1. INTRODUCTION

When an external fluid flows around bluff structures, such as chimneys, cables that support bridges, antennas, risers etc, an unstable wake is formed behind these structures and the vortices aft them starts to shed. These vortices sheds first from one side of the structure and then from the other side, generating oscillating surface pressures on the structure. If this structure is flexible or flexibly mounted, these oscillating pressures causes the structure to vibrate when the natural frequency of the structure and the frequency of vortex shedding synchronize, in a regime known as lock-in. This phenomenon is known as Vortex-Induced Vibrations (VIV), that can generate excessive loads on the structure and also fatigue damage. For these reasons, this subject has become the focus of many researches and engineers in the past few decades.

For VIV of a cylinder in the absence of a fixed wall, we have valuable works focusing on predict VIV using wake oscillator models with 1-DOF and 2-DOF (Skop and Balasubramanian (1997), Facchinetti *et al.* (2004), Ogink and Metrikine (2010), Kurushina *et al.* (2018)). Also, many experiments were conducted (Jauvits and Williamson (2004), Stappenbelt and Lalji (2008), Kang *et al.* (2016)). Finally, studies were also conducted in the filed of numerical simulations using Computational Fluid Dynamics (Martins and Avila (2019a), Martins and Avila (2019b), Khan *et al.* (2018)).

Subsea structures, such as pipelines used in oil and gas transmission and power cables, are widely used in the offshore industry. Due to the proximity of the seabed, the motion of these structures are different when compared to the motion of a structure in the absence of a boundary. Facing the importance of comprehending how these structures vibrates when exposed to a fluid flow, many experimental and numerical works were conducted in the past few decades with cylinders placed near a fixed wall.

In the last decades, many experimental studies were carried out to understand VIV of a rigid cylinder with different mass and damping ratios, and also different Reynolds number (Jauvits and Williamson (2004), Stappenbelt and Lalji (2008), Blevins and Coughran (2009), Kang *et al.* (2016)). In all of the mentioned experiments, the cylinders used are placed in the absence of a plane boundary (in this cases, the bottom of the testing flume or the wind tunnel), such that the boundary does not influence the observed VIV phenomena. Recent studies was also carried out using mathematical models based on wake oscillator, typically the van der Pol equation, to predict VIV of a rigid cylinder with 1-DOF and 2-DOF (Skop and Balasubramanian (1997), Facchinetti *et al.* (2004), Ogink and Metrikine (2010), Kurushina *et al.* (2018)). All of these works focus on solving a set of ordinary differential equations to evaluate the wake variable, commonly represented by q, that models the flow wake behind the cylinder. These studies do not have as main goal to reproduced the physics of the flow-cylinder interaction in detail, which could be achieved by solving the full fluid dynamic equations, as observed by Jin and Dong (2016). Computational Fluid Dynamics (CFD) has been proven to be the best approach to predict the dynamic characters of VIV, and some studies were carried out recently with this purpose (Martins and Avila (2019a), Martins and Avila (2019b), Khan *et al.* (2018)). Despite its already mentioned proven capability to predict VIV, CFD brings a important issue: their computational cost for simulations at realistic Reynolds numbers (Wu *et al.* (2012)).

(Bearman and Zdravkovich, 1978) experimentally analysed the flow around a cylinder with gap ratios, e/D, varying from $0 \leq e/D \leq 3.5$ and Reynolds number of $Re = 4.5 \times 10^4$. They observed that, as the gap ratio decreases, the vortex-shedding on the downstream region of the cylinder is suppressed. Also, they observed that the pressure distribution around the cylinder tends to be asymmetric as the gap ratio decreases.

(Barbosa *et al.*, 2017) conducted an experiment with a cylinder allowed to move in both in-line and cross-flow directions, with Reynolds number from $6500 \leq Re \leq 20000$ and gap ratios from $0 \leq e/D \leq 5$. They observed that, for $e/D \geq 2$, the boundary had no influence on the response of the cylinder. On the other hand, for $e/D < 2$ and as long as the cylinder do not touch the boundary, they observed that the amplitude of oscillations tends to decrease when the gap ratio decrease.

Recent experiments were conducted with the cylinder allowed to vibrate only in the cross-flow direction, as the works of (Bing *et al.*, 2009), (Hsieh *et al.*, 2016), (Daneshvar and Morton, 2020) and others.

From the review performed by the authors of the work, only two works considering wake oscillator model to predict VIV of a rigid cylinder near a fixed wall was found. The first one was proposed by (Jin and Dong, 2016). In their work, a wake oscillator model was used to study VIV of a cylinder allowed to move only in the cross-flow direction and initially placed near a fixed wall. The second one was proposed by (Barbosa *et al.*, 2017), in which a two degrees of freedom wake oscillator model was proposed, but during the simulations, the in-line movement of the cylinder was restrained, i.e., set to zero. This implies that the in-line movement of the cylinder have no influence on the cross-flow motion of the cylinder, which results in an one degree of freedom analysis, similarly to the one proposed by (Jin and Dong, 2016).

The absence of wake oscillator models to predict VIV of a rigid cylinder with two degrees of freedom placed near a fixed wall was the main motivation for the present work. Another important motivation is the importance of this phenomenon in the offshore industry.

2. MODEL DESCRIPTION

Consider a rigid cylinder with diameter D and mass per unit length m. This cylinder is exposed to a uniform fluid flow with density ρ and velocity V. Also, this cylinder is initially placed within a certain distance e from a fixed wall, as depicted in Fig. 1.

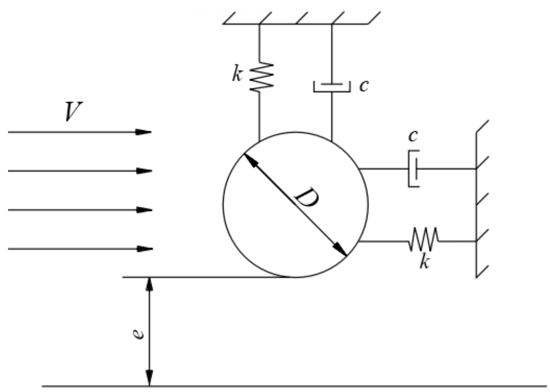


Figure 1: Schematic model of a cylinder with 2-DOF near a fixed wall.

As can be seen in Fig. 1, the cylinder is elastically mounted in the x and y directions through supports with stiffness k and damping coefficient c. The x axis defines the in-line direction, while the y axis defines the cross-flow direction. The governing equations to predict VIV of a rigid cylinder with 2-DOF near a fixed wall, in dimensionless form, are

$$\ddot{x} + 2\zeta\Omega_n\dot{x} + \Omega_n^2 x = \frac{C_{VX}}{2\pi^3 St^2(m^* + C_a)}, \tag{1}$$

$$\ddot{y} + 2\zeta\Omega_n\dot{y} + \Omega_n^2 y = \frac{C_{VY}}{2\pi^3 St^2(m^* + C_a)}, \tag{2}$$

$$\ddot{q} + \varepsilon(q^2 - 1)\dot{q} + q - \eta(e/D)\kappa\ddot{x}q = \beta(e/D)A\ddot{y}. \tag{3}$$

where overdots represents derivatives with respect to dimensionless time $\tau = \frac{t}{\omega_s}$, with t being the time and ω_s the vortex-shedding frequency of the cylinder. All the parameters of the model composed by Eqs. (1), (2) and (3) are presented in Table 1.

Table 1: Parameters of the model

Parameter	Definition	Unit
Dimensionless cross-flow displacement of the cylinder, y	$y = \frac{Y}{D}$	-
Dimensionless in-line displacement of the cylinder, x	$x = \frac{X}{D}$	-
Damping ratio of the cylinder in still water, ζ	$\zeta = \frac{c}{2\omega_n(m+m_a)}$	-
Natural frequency of the cylinder in still water, ω_n	$\omega_n = \sqrt{\frac{k}{(m+m_a)}}$	-
Added mass, m_a	$m_a = \frac{C_a\pi D^2 L}{4}$	kg
Potential added mass coefficient, C_a	$C_a = 1$ for circular cylinders	-
Length of the cylinder, L	Input parameter	m
Frequency ratio of the cylinder, Ω_n	$\Omega_n = \frac{\omega_n}{\omega_s}$	-
Vortex-shedding frequency, ω_s	$\omega_s = \frac{2\pi StV}{D}$	$\frac{rad}{s}$
Strouhal number, St	$St = \frac{\omega_s D}{V}$	-
Mass ratio of the cylinder, m^*	$m^* = \frac{m}{m_a}$	-
Wake variable, q	Unknown value of the wake oscillator equation	-
Tuning parameters, ε, κ and A	Obtained empirically	-
Empirical coefficients $\beta(e/D)$ and $\eta(e/D)$	Obtained empirically	-

The empirical coefficients $\beta(e/D)$ and $\eta(e/D)$ are added in the wake oscillator equation, Eq. (3), to account for the effects of the fixed wall on the motion of the cylinder. Analysing Eq. (3) it is possible to see that the empirical coefficients will acts as scalar factors of the cross-flow motion of the cylinder, multiplying $\frac{A}{D}\ddot{y}$, and of the in-line motion of the cylinder, multiplying $\frac{\kappa}{D}\ddot{x}q$.

The vortex force coefficients, C_{VX} and C_{VY}, are defined by (Qu and Metrikine, 2020) as

$$C_{VX} = (C_{DM}(1 - 2\pi St\dot{x}) + C_{VL}2\pi St\dot{y})\sqrt{(1 - 2\pi St\dot{x})^2 + (2\pi St\dot{y})^2} + \alpha C_{VL}^2(1 - 2\pi St\dot{x}) \mid 1 - 2\pi St\dot{x} \mid, \tag{4}$$

$$C_{VY} = (-C_{DM}2\pi St\dot{y} + C_{VL}(1 - 2\pi St\dot{x}))\sqrt{(1 - 2\pi St\dot{x})^2 + (2\pi St\dot{y})^2}. \tag{5}$$

where C_{DM} is the oscillating mean drag coefficient and C_{VL} is the vortex lift coefficient, defined as $C_{VL} = \frac{qC_{L0}}{2}$, with C_{L0} being the lift coefficient measured on a stationary cylinder. The hydrodynamic parameters utilized during all the simulations in the present work are listed in Tab. 2 and are the same used by Qu and Metrikine (2020).

Table 2: Hydrodynamic parameters utilized to simulate the VIV model. From (Qu and Metrikine, 2020)

Parameter	Value	Unit
Oscillating mean drag coefficient, C_{DM}	1.1	-
Lift coefficient measured on the stationary cylinder, C_{L0}	0.3	-
Drag coefficient measured on the stationary cylinder, C_{D0}	1.2	-
Coefficient to generate an oscillating mean drag coefficient of 1.1, α	2.2	-
Density of the water, ρ	1000	$\frac{kg}{m^3}$
Strouhal number, St	0.2	-

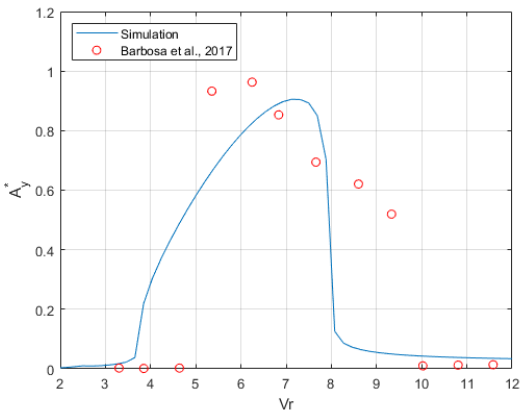
The experimental parameters used by Barbosa *et al.* (2017) are listed in Tab. 3 and were used to simulate the model of the present work.

Table 3: Parameters of the experiments performed by Barbosa *et al.* (2017).

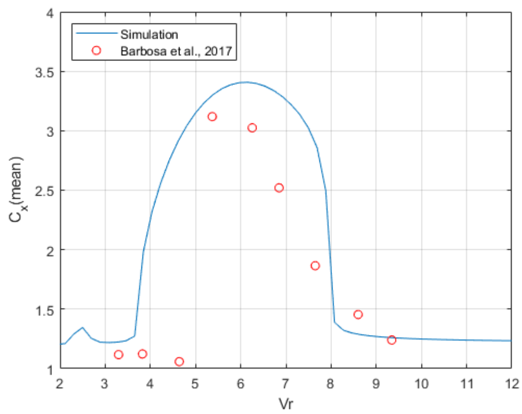
Parameter	Value	Unit
Diameter of the cylinder, D	0.04	m
Damping ratio, ζ	0.02	-
Natural frequency of the cylinder in still water, ω_n	8.17	$\frac{rad}{s}$
Mass ratio, m^*	5.5	-

3. CALIBRATING THE TUNING PARAMETERS OF THE WAKE OSCILLATOR MODEL

Before the calibration of the empirical coefficients, the tuning parameters, ε, A and κ, of the wake oscillator equation, Eq. (3), needs to be calibrated using experimental data. (Barbosa *et al.*, 2017) observed that, for $e/D > 2$ the influence of the fixed wall on the motion of the cylinder can be neglected. Thus, in the present work, a gap ratio of $e/D = 5$ is used to calibrate the tuning parameters of Eq. (3). Setting $\beta = 1$ and $\eta = 1$, the tuning parameters ε, κ and A are calibrated. The results presented in Fig. 2 are the cross-flow reduced amplitude, $A_y^* = A_y/D$, where A_y is the vibration amplitude in the cross-flow direction, and the mean drag coefficient, $C_x^{(mean)}$. All the results are in function of the reduced velocity, $V_r = \frac{2\pi V}{\omega_n D}$.



(a)



(b)

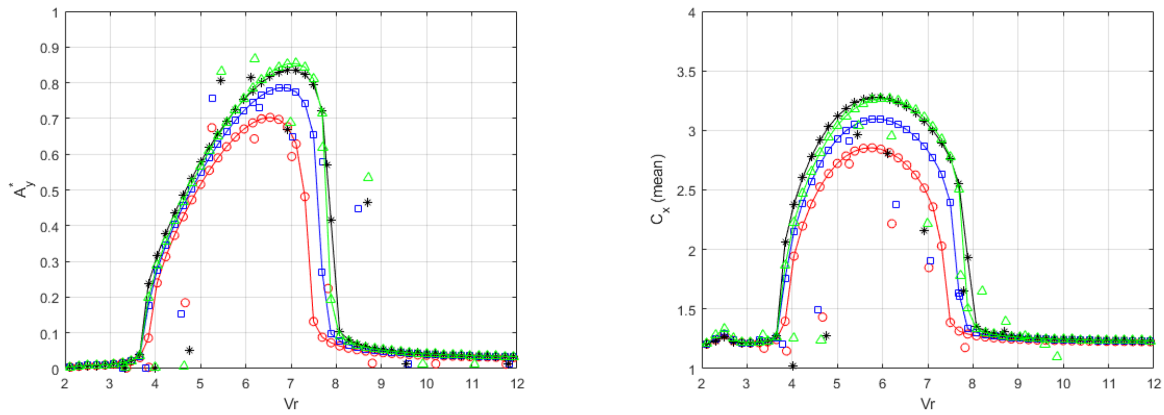
Figure 2: Results obtained by simulating the model with 2-DOF and experimental data from Barbosa *et al.* (2017) for $e/D = 5$. For (a) cross-flow reduced amplitude and (b) mean drag coefficient.

The tuning parameters used in all the simulations of the present work are, $\varepsilon = 0.1$, $A = 16$ and $\kappa = 3$.

4. CALIBRATION OF THE EMPIRICAL COEFFICIENTS OF THE MODEL

The next step is to calibrate the empirical coefficients of Eq. (3), β and η, to account for the effects of the fixed wall on the motion of the cylinder. For this purpose, the Eqs. (1), (2) and (3) are simulated using the parameters from Tabs. 2 and 3 and the experimental data from Barbosa *et al.* (2017). The gap ratios simulated are in the range of $2 \leq e/D \leq 0.75$. This range is used because, as already mentioned, for $e/D > 2$ the motion of the cylinder is no longer affected by the fixed wall and, for $e/D < 0.75$, the cylinder touched the bottom of the flume during the experiments performed by Barbosa *et al.* (2017). Due to that constant collisions, the signal of the mean drag coefficient, $C_x^{(mean)}$, was not considered reliable by the authors. Also, the typical motion of the cylinder inside the lock-in range is not present in the experimental result, because there is no symmetry in the signal obtained for the amplitude of vibration of the cylinder in the cross-flow direction, i.e., the values of maximum and minimum amplitudes of vibration are different.

The results presented in Fig. 3 are the cross-flow reduced amplitude, $A_y^* = A_y/D$ and the mean drag coefficient, $C_x^{(mean)}$ for $2 \leq e/D \leq 0.75$. All the results are in function of the reduced velocity.

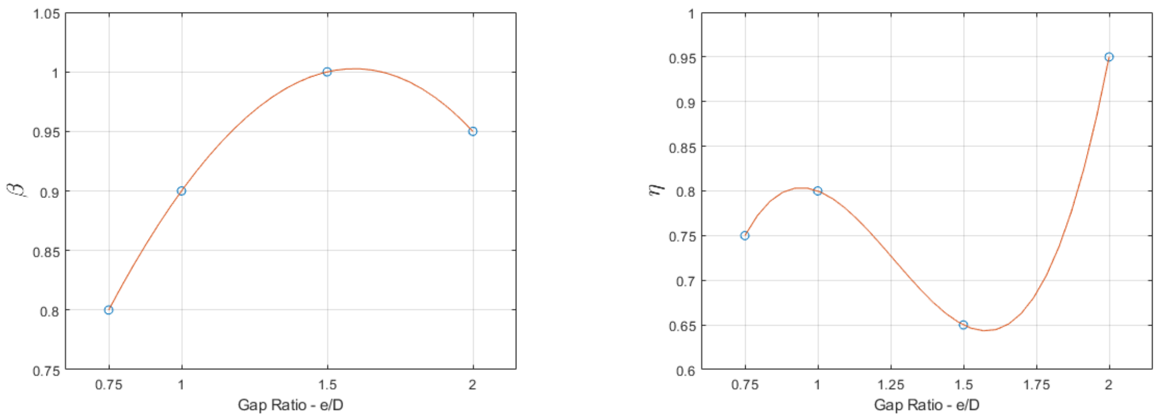


(a) (b)

Figure 3: Comparison with 2-DOF experimental data from Barbosa *et al.* (2017). Solid lines with symbols represents the simulations performed in this work in 2-DOF and experimental data are represented by symbols only. $e/D = 0.75$ ($\circ$), $e/D = 1$ ($\square$), $e/D = 1.5$ ($*$) and $e/D = 2$ ($\triangle$). For (a) cross-flow reduced amplitude and (b) mean drag coefficient.

The trend of the maximum cross-flow reduced amplitude, A_y^*, is clearly observed in Fig. 3a. As the gap ratio decreases, the maximum amplitude of vibration also decreases. This is due to the fact that, as the cylinder gets closer to the fixed wall, the shear layer in the downstream portion of the near wake is suppressed, leading to a non-fully development of the vortex shedding phenomenon. This affects the frequency of vortex shedding, ω_s, consequently affecting the lock-in range and maximum amplitude of vibration of the cylinder. For it being a 2-DOF analysis, it is expected that the same trend occurs for the mean in-line drag coefficient, $C_x^{(mean)}$. In fact, observing Fig. 3b, the same trend is visible.

For each gap ratio shown in Fig. 3, a set of empirical coefficients were manually setted by the authors of this work. After this process of calibration, using polynomial regression, the equations to calculate each empirical coefficient as function of the gap ratio was generated. Fig. 4 shows the curves of β and η as function of the gap ratio for $\leq 0.75e/D \leq 2$.



(a) (b)

Figure 4: Empirical coefficients (a) β and (b) η as function of the gap ratio.

The equations to calculate β and η as function of the gap ratio are:

$$\beta(e/D) = -0.0267(e/D)^3 - 0.18(e/D)^2 + 0.7767(e/D) + 0.33 \tag{6}$$

$$\eta(e/D) = 1.2533(e/D)^3 - 4.74(e/D)^2 + 5.5967(e/D) - 1.31 \tag{7}$$

5. DISCUSSIONS

In the present work, the response of a cylinder placed near a fixed wall was simulated using a new wake oscillator model with 2-DOF, developed from the modification of a wake oscillator model to predict VIV of cylinder in the absence of a fixed wall, where empirical coefficients are added to account for the effects of the fixed wall on the motion of the cylinder. As contribution, the coupled dynamic between the in-line and cross-flow vibrations is considered.

One new information are the equations to calculate the empirical coefficients as function of the gap ratio. As these equations do not depend on the mass and damping ratios of the cylinder, it could be use for any case. It is important to clarify that, for different values of mass and damping ratios, the appropriate procedure is to first calibrate the tuning parameters of the model with experimental data of a cylinder vibrating in the absence of a fixed wall and then calculate each empirical coefficient using Eq. (6) for $\beta(e/D)$ and Eq. (7) for $\eta(e/D)$.

It was observed by Roshko *et al.* (1975) that an attractive force appears on the cylinder as its gets closer to the fixed wall. This force tends to increases as the gap ratio decreases and, on the fixed wall, a repulsive force appears, and this force have the opposite direction of the motion of the cylinder, meaning that, when this repulsive force increases, as the cylinder gets closer to the bottom, the maximum amplitude of vibration of the cylinder decreases, as can be observed in Figs. 3a. When the cylinder touches the boundary, this force changes direction, now becoming an attractive force. The same trend is observed for the case in which two cylinder are arranged in-line or side by side, where this force appears in each one of the cylinders, and also increases as the gap between one cylinder to another decreases. As for the case of a cylinder near a fixed wall, when these cylinders collides, these forces also change its directions and become repulsive forces. This situation where cylinders are mounted near each other are of great importance in many engineering applications, such as tubes inside heat exchangers, power cable lines, risers etc, and a similar analysis could be conducted to generate a wake oscillator model to predict the response of the cylinder in these situations.

6. CONCLUSIONS

The present work had as principal goal to propose a new wake oscillator model with 2-DOF to predict VIV of a cylinder near a fixed wall. In general, the model was able to capture the main characteristics of VIV of cylinders near a fixed wall with $m^* = 5.5$ and $\zeta = 0.02$. The empirical coefficients, β and η, to account for the effects of the fixed wall on the motion of the cylinder were obtained by polynomial regression. This allowed the equations to calculate the empirical coefficients as function of the gap ratio to be of any order, and not only of order one, which is an advantage when compare to the model of Jin and Dong (2016), that considered that the empirical coefficients varies linearly with the gap ratio. Although, the empirical coefficients of the present work was considered to vary only with the gap ratio, e/D. For a case in which one would like to simulate the model for a different mass or damping ratios, it will be necessary to first set the tuning parameters from Eq. (3), ε, κ and A for $e/D \geq 2$ and then use the empirical coefficients presented in this work.

Moreover, the model developed is only capable to predict VIV of a circular cylinder with the same mass and damping ratios in both in-line and cross-flow directions.

When new experimental data with different gap ratios become available, it will be possible to refine the equations for the empirical coefficients. This will result in more precise model's responses for $m^* = 5.5$ and $\zeta = 0.02$, an important knowledge in the design of offshore structures near the seabed.

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8. DECLARAÇÃO DE RESPONSABILIDADE DAS INFORMAÇÕES

Eu, **Rafael Fehér**, portador do RG nº **45.767.491-4**, órgão experdidor **SSP** e do CPF **405.077.808-41** residente no endereço **Av. Estados Unidos, 93** Bairro: **Parque Das Nações**, Município: **Santo André - SP**. CEP: **09210-300**, assumo inteira responsabilidade pelas informações presentes nesse documento.